working paper
department
of economics

THE INFORMATIVENESS OF PRICES:
SEARCH WITH LEARNING AND
INFLATION UNCERTAINTY

Roland Benabou
Robert Gertner

No. 555 April 1990

massachusetts
institute of
technology
50 memorial drive
cambridge, mass. 02139

THE INFORMATIVENESS OF PRICES:
SEARCH WITH LEARNING AND
INFLATION UNCERTAINTY

Roland Benabou
Robert Gertner

No. 555 April 1990

THE INFORMATIVENESS OF PRICES: SEARCH WITH LEARNING AND INFLATION UNCERTAINTY

Roland Benabou
M.I.T.

Robert Gertner
University of Chicago

April 1990

This is a much revised and expanded version of "Inflation and Market Power in a Duopoly Model with Search." We would like to thank Olivier Blanchard, Dennis Carlton, Peter Diamond, Robert Lucas, Kevin Murphy, David Scharfstein, Jean Tirole, and Robert Vishny for beneficial discussions.

THE INFORMATIVENESS OF PRICES: SEARCH WITH LEARNING AND INFLATION UNCERTAINTY

Roland Benabou

M.I.T.

Robert Gertner

University of Chicago

ABSTRACT

Sources of aggregate cost uncertainty, such as real shocks or unanticipated inflation, reduce the information content of prices by making it difficult to separate relative and aggregate price variations. But when agents can acquire information, for instance by searching, such increased noise may in fact lead them to become better informed in equilibrium, and prices will reflect increased competition. To examine this issue, we develop a model of search with learning, in which consumers search optimally from an unknown price distribution, while firms price optimally given consumers' search rules. We show that the decisive factor in whether inflation variability increases or reduces the incentive to search, and thereby market efficiency, is the size of informational costs.

I. Introduction

Consider the problem of a consumer who observes an unexpectedly high price at a gas station. She must estimate how much of it is due to a common factor affecting all other suppliers, such as an oil shock or high inflation working its way through the economy; and how much of it reflects a specific supply or demand shock for this particular seller. If the first explanation is deemed more relevant, it is not worth looking for a better deal elsewhere; in the opposite case it may pay to do so. Similarly, when this consumer observes that an automobile manufacturer is offering large rebates on new cars, she must resolve whether this reflects factors which are specific to this particular brand (e.g. excess inventories) and make the offer a truly good deal, or whether the whole industry is perhaps having a sale. Again, her search behavior will vary with the inferences she draws from observed prices. These inferences are in turn based on her knowledge of the relative variability of idiosyncratic and aggregate shocks, inflation being one of the latter kind.

This problem of search with learning from prices is the main focus of the paper; it seems to us important on several counts. First, from a microeconomic point of view, the inferences which buyers draw from prices underlie their demand functions, and should therefore play an important role in determining how markets react to oil shocks, weather variations, technological innovations, etc. Secondly, from a macroeconomic point of view, it has implications for what is often thought to be an important source of welfare losses from unanticipated inflation: a deterioration in the information content of prices, operating through increased relative price variability. [1] According to this view, stochastic inflation

---

1. Several studies documenting the correlation between the variance of unanticipated inflation and relative price variability (Vining and Elwertowski (1976), Parks (1978), Fischer (1981)) seem to lend support to this idea. Hercowitz (1982), on the other hand, finds that aggregate real shocks, not monetary shocks, explain relative price dispersion in the United States. As the average rate and the variability of inflation are also correlated (e.g. Taylor (1981), Pagan, Hall and Trivedi (1983)), high inflation is often seen as indirectly responsible for any informational costs of unanticipated inflation. The empirical literature on these issues is surveyed extensively in Fischer (1981) and Cukierman (1983).

constitutes a source of aggregate noise in market signals, leading to inefficient allocation decisions. While this is well understood in models where the information structure is exogenous (Lucas (1973), Barro (1976), Cukierman (1979)), what happens when agents can decide to acquire additional information, by searching or otherwise, has not been explored. Finally, another common, but not yet formalized idea, is that sellers can "hide" behind aggregate or inflationary noise to charge higher real prices, taking advantage of consumers' reduced information to increase their markups. [2]

In this paper we attempt to explore these issues by analyzing the effects of aggregate cost uncertainty, arising from real shocks or from unexpected general inflation, on the efficiency of allocations in a market with endogenous information-gathering and price-setting.[3]

The literature on the effects of inflation on the information content of prices has traditionally focused on prices' role as signals for producers rather than consumers. Misperceptions by producers concerning aggregate and relative price movements (Lucas (1973), Barro (1976), Cukierman (1979), Hercowitz (1981)) or permanent and transitory movements in relative prices (Cukierman (1982)) generate inefficiencies as well a as relationship between the variance of inflation in the price level and that of relative prices. The first type of misperceptions is often dismissed as implausible on the grounds that the availability of macroeconomic price or monetary statistics should solve the problem. This criticism takes the informational structure of these models too literally, rather than as a simplifying device to make the point that inflationary noise interacts with the functioning of

2. For instance, whether or not oil companies and gasoline retailers engaged in such behavior following the shutting down of the Alaskan pipeline after the "Exxon Valdez" oil spill has been the subject of intense debate.

3. For a related analysis of the effects of anticipated, or trend inflation, see Benabou (1988), (1989).

the price system. A more valid, and constructive restatement of this criticism consists in pointing out that agents can indeed acquire information, but at some cost; the clearest example of this is search. This in turn has two important consequences:

(i) Sellers generally have market power.

(ii) Information is endogenous: information acquisition and pricing strategies are determined jointly in equilibrium, and both depend on inflation.

Our paper is in the "misperceptions" or "learning" tradition but emphasizes the informational problems of consumers as well as producers, and embodies the above remarks, on which we now elaborate.

First, informational costs realistically imply that sellers have market power. While growing more sophisticated in its treatment of signal-extraction, the "misperceptions" literature has maintained the assumption of perfect competition, although it seems at odds with that of imperfect price information. Indeed, their coexistence relies on a drastic asymmetry between informational costs within a market (zero) and across markets (infinite). In modern economies, no market is really cut off from the others in terms of information, and it is more likely that both types of costs are of the same order of magnitude. A realistic information-based model should therefore probably recognize the presence of some (possibly small) monopoly power . As a result, inflationary uncertainty may alter relative prices, but these are not Walrasian to start with; the issue then becomes whether preexisting distortions will be worsened or alleviated.

Secondly, in the absence of fully adequate price statistics, buyers and sellers have an incentive to seek information, for instance by searching (within a market) or just by checking other prices (across markets). An equilibrium analysis of inflationary misperceptions should therefore embody the effects of the inflation process on this incentive to acquire information. For instance, we shall see that a deterioration in the quality of price signals due to increased

inflationary noise can lead agents to acquire more information, making them actually better informed in equilibrium. Naturally, the costs of acquiring information (e.g. search costs) will play a determinant role.

To take account of these points, we build a model with a stochastic structure somewhat similar to those of Lucas (1973) and Barro (1976), but where information can be acquired endogenously and the market structure is very different. Duopolistic firms observe their own production costs, then set prices. Since costs are correlated, buyers learn from one firm's price something about the price of the other. Given these inferences and their knowledge of firms' strategies, they decide whether or not to search. Conversely, when setting their prices firms take account of buyers' search rule, as well as of their own inferences about their competitor's cost and price.

This requires addressing the more general problem of search market equilibrium with Bayesian learning. To our knowledge this paper, and independent work by Dana (1990) represent the first models which combine optimal adaptive search (e.g. Rothschild (1973), Rosenfield and Shapiro (1981)) with equilibrium pricing. Not surprisingly, the problem is quite complex, and this is why we have to focus attention on a single market, as opposed to a general equilibrium framework. The question then arises of how to capture, in a microeconomic setting, the idea that inflation reduces the informativeness of prices. We identify an increase in inflation uncertainty with an increase in the variance of an industry-wide cost shock. In effect, we take as given the fact that inflation affects inter-industry costs, and examine how such shocks affect intra-industry pricing behavior and market performance.

Of course, this is only a partial and crude representation of inflation. First, inflation should affect the demand side as well as the supply side. But this is essentially an issue of timing: realistically, when consumers see prices change unexpectedly, their resources have not

yet been fully and unambiguously affected by the inflationary shock (if they had, the price change would not be unexpected, and there would be no difficulty in assessing relative prices). Similarly, when firms discover unexpected changes in costs, they have not yet experienced the full increase in nominal demand which consumers will eventually address to them. Secondly, our model has no money. However, if one takes as given that inflation affects nominal costs, there is no substantial problem with calling the numeraire good money. Since all price changes are unexpected by agents given their information (any inflationary trend has already been factored out), the dollar prices which they observe are their best assessments of real prices, and may enter their utility and profit calculations without implying any money illusion.

Thus in spite of the model's obvious limitations, we feel that what we learn from it about the effects of real, aggregate cost shocks remains relevant for genuine, money-driven inflation. The reader who does not share these convictions can maintain a purely "real" reading of the paper; indeed the relationship between the stochastic structure of supply shocks and market efficiency is of interest independently of any possible link to inflation.

A well-known discussion of the relationship between consumer search behavior and macroeconomic shocks is contained in Okun (1981). He argues informally that search and the repeat purchase behavior of consumers may reduce the adjustment of prices to demand shocks. The reasoning is that a firm's previous customers will only search if the original firm raises its price. Lowering price can only attract consumers who are good shoppers and therefore have high demand elasticities. The argument, although intuitively plausible, lacks a justification for consumers' search behavior. In particular, if inflation is expected, a nominal price increase at the inflation rate should not induce search.

In our model, where search and pricing decisions are optimal given the stochastic structure of costs, the variability of the joint cost shock will affect consumers' signal-extraction problem, hence their search rules. This in turn will determine the elasticity of demand faced

by each firm, hence its pricing strategy and ultimately social welfare. The relationship between aggregate cost or inflation uncertainty, monopoly power and market efficiency created by this mechanism is the main focus of the paper.

We identify two major effects of an increase in the variance of inflationary shocks. We refer to the first one as the correlation effect. It operates so that if search costs are sufficiently high, an increase in the variability of inflation works to increase market power and real prices, while if search costs are sufficiently low, an increase in the variability of inflation works to decrease market power and real prices. Intuitively, the argument is the following. As the variance of the joint cost shock increases, the correlation between firms' costs increases. Since optimal prices depend positively upon cost, prices also become more correlated. This leads Bayesian consumers to put more weight on the first observed price and less on their prior, when forming their posterior belief about the price distribution at the second firm. Thus, if the observed price is high, greater variance of the joint cost shock implies a higher conditional expectation of the price at the second firm. This reduces the value of search for a given price at the first store, as it becomes less likely that its high price is really a bad deal. Conversely, if the observed price is low, greater variance of the joint cost shock implies a lower conditional expectation of the price at the second firm. This increases the value of search, as even this low price is less likely to be a really good deal. Now if search costs are low, buyers' reservation price is low, and therefore through the correlation effect inflation variability tends to decrease it further, thereby making the market more competitive. Conversely if search costs are high, so is buyers' reservation price, and inflation variability will tend to increase it even more, and with it firms' market power.

An increase in the variance of the joint cost shock affects not only the correlation of costs, but also their unconditional variance, and generally the conditional variance as well. This in turn will tend to increase the conditional variance of prices, and thereby the option

value of search, as is well known. This effect, to which we refer as the variance effect, tends to raise the value of search and lower firms' market power, as the variance of the inflationary shock increases.

We identify several other channels through which aggregate cost shocks affect market performance, but the correlation and variance effects stand out as both more important and general. They formalize and fill the gaps in the intuition one generally has about the impact of inflation uncertainty on buyers' incentives to search.

The main conclusion of the paper is that when it is recognized that informational imperfections generate imperfect competition and endogenous information gathering, the a priori case for information-related welfare losses from variable inflation is much weaker. Indeed, inflation has many effects (which we identify) on the price system, several of which tend to improve market efficiency. We show that whether inflation lowers or raises welfare crucially depends on how costly it is to acquire information. In analyzing the interaction of anticipated inflation and search, Benabou (1989) comes to quite similar conclusions. Thus, whether through its trend or its variance, inflation still seems to have no single identifiable efficiency cost which could be expected to swamp the benefits across all market structures and inflationary regimes. The strong aversion to inflation which people almost universally proclaim remains hard to reconcile with the consequences of the sophisticated behavior and information utilization with which economic models endow them.

The paper proceeds as follows: Section II describes the model and Section III contains the construction of equilibrium strategies. The effects of changes in inflation variability are discussed in Section IV, both through analytic examples and a number of simulations. Section V concludes.

II. The Model

In this section and the next we present a model of a duopolistic search market equilibrium with Bayesian learning. Buyers' search decisions depend on the inferences they make from observed prices, taking into account firms' strategies. Conversely, firms' pricing decisions incorporate their own inferences about their competitors' prices, and their knowledge of buyers' inference and search rules.

1. Market and information structure. There are two identical firms, with constant marginal costs c_1 and c_2, which are drawn from a symmetric joint distribution with support $[c^-, c^+] \times [c^-, c^+]$, $0 \leq c^- < c^+ < +\infty$. For instance, each firm's cost c_i could be the sum or product of a common cost shock θ (e.g. inflation) and a private cost shock γ_i (real cost). Firm i observes its own cost c_i, but not its rival's cost c_j, although c_i provides information about c_j.[4] Buyers do not observe cost realizations but the joint distribution of costs is common knowledge. We denote by $F(c_2 | c_1)$ and $f(c_2 | c_1)$ the distribution and density of firm 2's cost, conditional on firm 1's. We assume that $f(c_2 | c_1)$ is continuously differentiable in both c_1 and c_2 almost everywhere, and that costs are positively correlated in the following sense:

$$F_2(c_2 | c_1) \equiv \frac{\partial F(c_2 | c_1)}{\partial c_1} \leq 0, \quad \forall c_1, \tag{1}$$

$$\frac{\partial}{\partial c_1}\left[\frac{f(c_2 | c_1)}{1 - F(c_2 | c_1)}\right]_{c_2 = c_1 - 0} \leq 0, \quad \forall c_1. \tag{2}$$

4. Suppose for instance that $c_i = \theta + \gamma_i$, $i = 1, 2$; then c_1 contains information about c_2 even though we assume that firm 1 does not know θ, but only c_1. This assumption is not essential but fits well with the idea of aggregate uncertainty. Firms are themselves buyers of inputs, and do not have perfect information on whether high or low materials and labor prices are specific to their own suppliers or economy-wide. In a richer model, of course, firms could also decide to become informed at some cost.

Condition (1) is just first-order stochastic dominance; (2) also means that a higher c_1 signals a higher c_2. To interpret it, suppose first that it were required at every (c_1, c_2). It would then say that the hazard rate for finding the true c_2, as one moves from c^- to c^+, is decreasing in c_1. It is easily shown that such will be the case if $f(c_2 \mid c_1)$ has the usual monotone likelihood ratio property, i.e., if $f(c_2' \mid c_1)/f(c_2 \mid c_1)$ increases in c_1 for any $c_2' > c_2$. But (2) need only hold as c_2 tends to c_1 from below, so it is in fact quite weak.[5] Condition (2) will ensure that a firm's equilibrium expected profit function (conditional on its own cost) is quasi-concave.

We now turn to buyers. There is a continuum of identical consumers, with measure normalized to one. Let $S(p) = \int_p^\infty D(r)dr$ be the surplus each of them derives from buying at p, where $D(p) = -S'(p)$ is her demand function in the absence of search. We make the standard assumption that a firm's profit per customer $\Pi(p, c) \equiv (p-c)D(p)$ is strictly quasi-concave in p for all c in $[c^-, c^+]$. Since $\Pi(p, c)$ is the profit function of a monopolist with cost c, let $\Pi_m(c)$ denote its maximum value, achieved at $p_m(c)$.

2. <u>Search and learning from prices</u>. Initially, half the buyers observe firm one's price and half firm two's price, at no cost. Given the observed price, they must decide whether to purchase or to search and find out the other firm's price. Searching entails a cost σ but allows the consumer to buy at the cheapest of the two prices. The assumption that the first offer can be recalled costlessly means that σ is a pure informational cost, rather than a transportation or communication cost.

Given the first observed price, say p_1, a consumer must first infer the extent to which

---

5. Note also that (1) tends to make (2) hold, and that if an increase in c_1 leads to a lower probability that c_2 is in $[c^-, c_1]$, i.e. $f_2(c_2 \mid c_1) \leq 0$ for $c_2 < c_1$, then (2) holds.

it reflects a firm-specific shock or a joint shock. From this inference she then forms posteriors about the other firm's price, with distribution $G(p_2 \mid p_1)$ and density $g(p_2 \mid p_1)$ on the price support $[p^-, p^+]$. Finally, given these beliefs, the consumer will decide that it is worth finding out p_2 before buying if the expected benefit from search,

$$W(p_1) \equiv \int_{p^-}^{p_1} [S(p_2) - S(p_1)] g(p_2 \mid p_1) dp_2 = \int_{p^-}^{p_1} D(p_2) G(p_2 \mid p_1) dp_2, \quad (3)$$

is larger than the search cost σ; otherwise she will just buy right away at p_1.

Note from (3) that observing a high price has two effects on the expected return from search. For a given distribution, a higher p_1 makes it more likely that a better deal can be found, and this tends to increase $W(p_1)$; but if firms' prices are correlated, a high p_1 is "bad news" about the distribution $G(p_2 \mid p_1)$ of the other firm's price, and this tends to reduce $W(p_1)$. As is well-known from the literature on optimal search from an exogenous unknown distribution (DeGroot (1970), Rothschild (1973), Rosenfield and Shapiro (1981)) this learning effect can result in search strategies where a price p_1 is rejected but a higher price $p_1' > p_1$ is accepted. To preserve the reservation price property, Rothschild (1973) and Rosenfield and Shapiro (1979) make assumptions on the distribution of prices (equivalently $G(p_2 \mid p_1)$) and on consumer preferences ($D(p) \equiv 1$) which ensure that the learning effect is not too strong, so that $W(p_1)$ is monotone. Alternatively, Rosenfield and Shapiro impose the condition that the return to an additional search never cross the horizontal line $W(p_1) = \sigma$ from above. In an equilibrium model, however, $G(p_2 \mid p_1)$ incorporates firms' optimal pricing rule, therefore no such assumptions can be made. Moreover, the pricing rule itself depends on buyers' inferences, making the problem quite complicated.

We shall nonetheless follow similar lines of reasoning, but with respect to the exogenous distribution of costs $F(c_2 \mid c_1)$. We mainly focus on equilibria where buyers' search rules

have the reservation price property (reservation price equilibria), and show that a pure strategy reservation price equilibrium exists provided that either:

i) Firm's costs are not too correlated (F_2 is not too large),

ii) Buyers' search cost σ is relatively small.

The first assumption will ensure that $W(p_1)$ is monotonic.[6] Alternatively, the second will ensure that $W(p_1)$ never falls below σ once it has risen above.

The model also builds on Reinganum (1979). The two essential differences are that we analyze a duopoly while she assumes a continuum of firms, and we have correlated costs while she has independent costs. These two features allow respectively for search to take place in equilibrium, and for consumers to learn from prices. We expand on these remarks in Section III.

3. <u>Technical conditions</u>. This paragraph gathers additional, more technical assumptions. Since they offer little intuition, some readers might want to go directly to Section III.

First, in addition to (1) and (2), the distribution of costs must satisfy:[7]

$$f(c \mid c) > 0, \quad \forall c \in [c^-, c^+], \tag{4}$$

Note that this implies $F(c \mid c) > 0$, for all $c > c^-$.

Turning now to the demand side, we assume that $-D'(p) = S''(p)$ is bounded on

6. Consider a parameterized family of conditional distributions $F^\lambda(c_2 \mid c_1)$ with partial derivatives $F_1^\lambda = f^\lambda$ and F_2^λ which are continuous in λ, and such that for $\lambda = 0$, $F_2^\lambda = 0$. Then (i) holds for λ not too large.

7. We define $f(c^- \mid c^-)$ and $f(c^+ \mid c^+)$ as $\lim_{c \to c^-} f(c \mid c)$ and $\lim_{c \to c^+} f(c \mid c)$, respectively. These limits can be positive even with $f(c^- \mid c_1) = 0$ or $f(c^+ \mid c_1) = 0$, $\forall c_1 \in (s^-, c^+)$.

$[c^-, c^+]$, i.e. $\Delta \equiv \sup\{-D'(p) \mid c^- \leq p \leq c^+\} < \infty$. This may require $c^- > 0$. Finally, the strict quasi-concavity of $\Pi(\cdot, c)$ implies that:

$$\rho(p,c) \equiv \frac{\Pi_p(p,c)}{p_m(c)-p} \quad > \quad 0, \qquad \forall p, \quad c \leq p < p_m(c).$$

This also holds in the limit at $p_m(c)$, since $\Pi_{pp}(p_m(c),c) < 0$. Therefore, by uniform continuity on the compact set $K \equiv \{(c,p) \mid c^- \leq c \leq c^+, c \leq p \leq p_m(c)\}$:

$$0 < m \equiv \min\{\rho(p,c),(c,p) \in K\} \leq \max\{\rho(p,c),(c,p) \in K\} \equiv M < \infty. \tag{5}$$

We shall assume:

$$\Pi_m(c^-) \;\; \geq \;\; S(c^-) - S(c^+) + M\left(\frac{\Delta}{m}\right)^2 (c^+ - c^-)^2. \tag{6}$$

This basically requires that the monopoly profit functions $\Pi(p,c)$ be neither too flat nor too spiked, that monopoly profits for the most efficient firm be sufficiently large, and that the range of possible costs (c^-, c^+) not be too wide. Condition (6) will ensure the existence of a solution to the differential equation defining the optimal price strategy of firms with relatively high costs.

intuitive conditions. We do not have conditions for when the mixed strategy equilibrium exists, or a very general existence theorem. Nor do we rule out the possibility that for some parameter values there exists more than one of the four equilibrium types, or even some other, less intuitive type.[8]

The first proposition shows that all equilibria share an intuitive feature: low cost firms charge their monopoly price.

Proposition 1: In any equilibrium of the game, there exists an $\epsilon > 0$ such that a firm with cost $c \in [c^-, c^- + \epsilon]$ sets price equal to $p_m(c)$.

Proof: See appendix.

The intuition is quite simple. Since search costs are strictly positive, consumers who observe a price sufficiently close to the lowest price charged in equilibrium will not search. If the lowest price charged in equilibrium is less than $p_m(c^-)$ the firm charging this price can deviate and raise its price without losing any customer, thereby increasing its profits. The argument if the lowest price is above $p_m(c^-)$ is even simpler. Given Proposition 1, it will be useful to define:

$$V_m(c) \equiv \int_{c^-}^{c} [S(p_m(c_2)) - S(p_m(c))] f(c_2 \mid c) dc_2 = \int_{c^-}^{c} D(p_m(c_2)) p_m'(c_2) F(c_2 \mid c) dc_2 . \quad (7)$$

8. For instance, one can not even exclude equilibria where a firm's price $p(c_1)$ decreases with its cost c_1 over some range. The usual revealed preference argument fails here because c_1 affects firm 1's expected demand function through its correlation with c_2 and p_2. We shall, however, restrict attention throughout the paper to equilibria where $p(c)$ is non-decreasing.

III. Equilibrium

1. General properties. We look for a symmetric, perfect Bayesian equilibrium of the game between firms and buyers, with a consistency restriction on beliefs off the equilibrium path (see Fudemberg and Tirole [1989]): a consumer's observation of firm 1's price p_1 only directly affects her beliefs about firms 1's cost c_1. Thus, if p_1 is off the equilibrium path, there are no restrictions on the consumer's beliefs about c_1; but in determining whether or not to search, she must use these arbitrary beliefs about c_1 to form beliefs about c_2 which are consistent with the joint distribution of costs and Bayes' rule. Moreover, these beliefs about c_2 and the equilibrium strategy must be used to form beliefs about p_2. This will be important below; for example, if a consumer observes at firm 1 a price less than the lowest price p^- played with positive probability, she must still put zero probability on the other firm's having a price less than p^-.

We identify four different types of equilibria to the model. If search costs are sufficiently high, each firm will be able to charge its monopoly price without triggering any search. If search costs are not quite large enough to support this outcome, there may still be an equilibrium without search, where firms with higher costs bunch at consumers' reservation price to prevent search. These two equilibria are qualitatively similar to those of the Reinganum model. A new type of equilibrium arises for lower search costs: a pure strategy, reservation price equilibrium with buyers searching at higher prices and firms' markups decreasing to zero as their costs increase toward the maximum c^+. The fourth possible type of equilibrium involves mixed strategies. High cost firms charge prices which make consumers indifferent between buying and searching, and the fraction of consumers which search at any price makes this pricing rule optimal for firms.

We prove the existence of the first, second or third types of equilibria, under quite

$V_m(c)$ is the value of search $W(p)$ when observing a price $p = p_m(c)$, if all firms with cost below c charge their monopoly price, and no firm with cost above c charges less than $p_m(c)$. We now move to a characterization of equilibrium, starting with the case of large search costs.

2. Monopolistic equilibrium. When search costs are large enough, the range of monopolistic pricing of Proposition 1 can cover all cost realizations, and consumers will still not search, independently of the price observed at the first store.

Proposition 2: If $V_m(c) < \sigma$ for all c in $[c^-, c^+]$, there exists an equilibrium in which each firm charges its monopoly price $p_m(c)$, and consumers never search.
Proof: See appendix.

This is quite intuitive. No firm can earn more per customer by deviating, nor can it attract more than half of the customers, since none of them search. Conversely, not searching is optimal given the definition of V_m.

3. No-search equilibrium with bunching. When search costs are not large enough to support the equilibrium of Proposition 2, there may still be an equilibrium in which no consumers search. Define c^* as the smallest solution to:

$$V_m(c^*) \equiv \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 | c^*)dc_2 = \sigma . \tag{8}$$

and let $p^* = p_m(c^*)$. A consumer is indifferent between search and purchasing at a store charging p^*, if all firms with cost $c \leq c^*$ charge $p_m(c)$ and no firm with cost $c \geq c^*$ charges less than p^* (so that p^* reveals c^*). We shall focus on reservation price equilibria where consumers accept prices p_1 up to p^* but reject higher ones.[9]

Firms with $c \leq c^*$ are still able to charge their monopoly price. Consider, however, a firm with cost just above c^*. If it charges its monopoly price, it will induce search; rather than accept the resulting first-order loss in customers (they search and find a lower price with a probability of at least $F(c \mid c)$), it prefers charging p^*, which causes no loss of customers and only a second-order effect on profits per customer. In fact, we show that if $p^* > c^+$, there is an equilibrium in which <u>all</u> firms with cost above c^* charge p^*. In this equilibrium, consumers do not search but prices are constrained by the possibility of search.

<u>Proposition 3:</u> If search costs σ are such that $p^* > c^+ > c^*$, there exists an equilibrium in which consumers have reservation price p^* and firms' pricing rule is: $p(c) = p_m(c)$ for $c \leq c^*$ and $p(c) = p^*$ for $c^* < c \leq c^+$.

<u>Proof:</u> See appendix.

To sustain this equilibrium, we can simply assign beliefs which make it profitable to search, in response to any price $p_1 > p^*$; for instance, $c_1 = c^*$. Any firm which deviates to such a price then earns zero profits, while it could earn positive profits by playing its

---

9. It can be shown that any reservation price equilibrium with $p(c)$ non-decreasing must have the same form as those we examine (with p^* simply replaced by $\hat{p} \leq p^*$); moreover those with $\hat{p} < p^*$ necessarily rest on very implausible out-of-equilibrium beliefs. Since the basic features of the equilibrium and the spirit of the results remain unchanged, we do not think it worthwhile to go into the complexities of equilibrium refinement, and simply concentrate on the more natural reservation price equilibrium where $\hat{p} = p^*$.

equilibrium strategy. Therefore such deviations will not occur, and this justifies the imposed beliefs. Each firm chooses instead the price $p \leq p^*$ which maximizes its profit per customer and prevents search. Finally, by definition of c^*, accepting offers below p^* is optimal for consumers.

The equilibria in Propositions 2 and 3 are analogous to those of the Reinganum (1979) model, except that consumers' reservation price p^* depends here on the learning which occurs in equilibrium.

4. Reservation price equilibrium with search. Smaller search costs lead to a new but much more difficult case, in which consumers' reservation price p^* is less than c^*. A firm whose cost exceeds p^* can then not avoid search, unless it makes negative profits. If the market contained a continuum of firms (as in Reinganum's model), such a firm would have to stay out, because its consumers who searched would all find a lower price. However, with only two firms in the market (more generally a finite number), it is possible to charge a price which induces search, but still expect positive profits when one's rival, who follows the same strategy, has an even higher cost, hence an even higher price.

We now characterize a pure strategy reservation price equilibrium in which search actually takes place. Firms' pricing strategy is illustrated on Figure 1. For low cost realizations, it is similar to that of Proposition 3: a firm with cost below c^* charges $p_m(c)$ and a firm with cost between c^* and some $c^s > c^*$ charges consumers' reservation price $p^* = p_m(c^*)$. A firm with higher cost realization c, however, charges a price $p_F(c) > p^*$, so all consumers who visit this firm first search.

We now derive $p_F(c)$. If firm i charges a price p_i which induces search, it will sell to all consumers if its rival has a higher price, and to none if its rival has a lower price. Therefore

EQUILIBRIUM PRICING STRATEGY WITH SEARCH

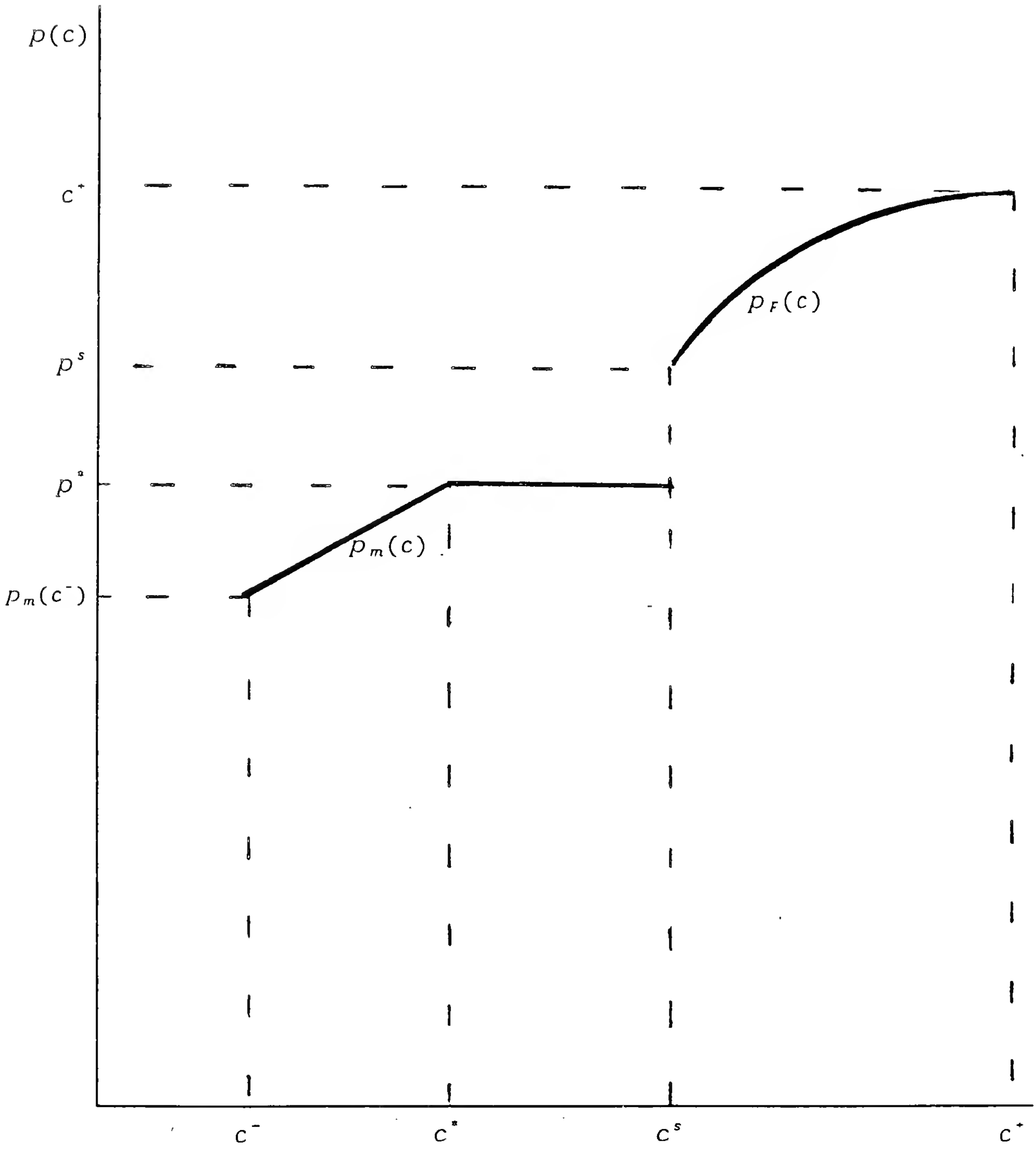

Figure 1

firm i's probability of making sales is the probability that firm j has a higher price, conditional on firm i's true cost c_i, and its expected profits are:

$$\Psi(p_i, c_i) = \Pi(p_i, c_i) \cdot [1 - Prob[p(c_j) \leq p_i \mid c_i))], \tag{9}$$

where $p(\cdot)$ is the equilibrium pricing strategy. Assume for now that $p(\cdot)$ is increasing and differentiable on $[c^s, c^+]$, with $p(c^s) > p^*$; this will be verified below. We can then rewrite:

$$Prob[p(c_j) \leq p_i \mid c_i] = F[p_F^{-1}(p_i) \mid c_i]$$

and differentiate (9) with respect to p_i to obtain the first-order condition which $p(c)$ must solve:[10]

$$p'(c) = \frac{f(c \mid c)}{1 - F(c \mid c)} \cdot \frac{\Pi(p(c), c)}{\Pi_p(p(c), c)}. \tag{10}$$

This differential equation must be satisfied along the price path in the region of costs $[c^s, c^+]$ which lead to search. The boundary condition is found by considering a firm with the highest possible cost, c^+. Such a firm makes no sales since consumers search and are sure to find a lower price. Therefore, it must be the case that $p(c^+) = c^+$; otherwise the firm would lower its price by a little bit and make positive expected profits, since it would make sales at a price above cost whenever its rival's price was higher. Because $p(c^+) = c^+$, (10) does not satisfy Lipschitz conditions at (c^+, c^+),[11] so standard theorems are not applicable. Instead, we construct the solution as the fixed-point of a contraction mapping; this is where condition (6) is needed.

---

10. Since charging $p > p^*$ leads consumers to become fully informed, it is not surprising that this pricing rule is quite similar to the optimal bidding rule in an auction with correlated values (Milgrom and Weber [1982]). The difference, and source of difficulty, is that $\Pi(p, c)$ cannot be expressed as a function $U(p - c)$.

11. Nor at points $(p_m(c), c)$.

Lemma 1: The differential equation (10) on (c^-, c^+), with terminal condition $p(c^+) = c^+$ has a unique solution $p_F(c)$, satisfying $c \leq p_F(c) < p_m(c)$, with equality only at c^+. Moreover, $p_F{}'(c) > 0$, for all c in $[c^-, c^+]$.

Proof: See appendix.

While $p_F(c)$ solves the first-order condition (10), it remains to prove that it really characterize optimal prices for $c \geq c^s$. First, using condition (2) on the distribution of costs, we show that equilibrium profits are strictly quasiconcave.

Lemma 2: Assume that buyers have reservation price $p^* = p_m(c^*)$. If firms with cost above some level $c^s > c^*$ charge $p_F(c)$, while firms with cost $c \leq c^s$ charge $\min(p_m(c), p^*)$, then a firm's profits from charging any price $p \geq p^s$ are:

$$\Psi(p,c) \equiv \Pi(p,c).[1 - F(p_F^{-1}(p) \mid c)]. \tag{11}$$

They are strictly quasiconcave and maximized over $p \in [p^s, c^+]$ at $p = p_F(c)$.

Proof: See appendix.

The final step in characterizing firms' strategies is to find the threshold cost c^s separating those which prefer to charge p^* from those which prefer to charge $p_F(c) > p^*$. The first strategy prevents search but the second yields greater profit per consumer, if they come back. At c^s a firm is indifferent between the two, so c^s is defined by:

$$\left[1 - \frac{1}{2}F(c^s \mid c^s)\right] \cdot \Pi(p^*, c^s) = [1 - F(c^s \mid c^s)] \cdot \Pi(p_F(c^s), c^s). \tag{12}$$

The left-hand side represents profits from charging p^*. A firm which does this sells to all consumers who visit it first (1/2) and to all consumers who visit the other firm first and

observe a price above p^*. Given the symmetry of the equilibrium pricing strategy, this is just 1/2 times the probability that the other firm has cost above c^s, which is $\frac{1}{2}[1 - F(c^s \mid c^s)]$. The right-hand represents profits from charging $p_F(c^s)$. A firm which does this sells to all customers who visit it first, search, find a higher price and return, which given symmetry, is $\frac{1}{2}[1 - F(c^s \mid c^s)]$ customers. It also sells to customers who visit the other firm first, search, and find a lower price, which is also $\frac{1}{2}[1 - F(c^s \mid c^s)]$ customers.

Lemma 3: Assume that buyers have reservation price $p^* = p_m(c^*)$. There exists a unique $c^s \in (c^s, c^+]$, such that the following strategies are mutual best responses for firms:

(i) For $c \in [c^-, c^*]$, $p(c) = p_m(c)$;

(ii) For $c \in [c^*, c^s)$, $p(c) = p^*$;

(iii) For $c \in [c^s, c^+]$, $p(c) = p_F(c)$.

Moreover, c^s and p^s are continuous and non-decreasing in c^*.

Proof: See appendix.

This result concludes the characterization of firms' strategies: if there exists a pure strategy equilibrium where consumers have reservation price p^*, it is uniquely defined by c^*, $p_F(\cdot)$, and c^s, which are the unique solutions to (8), (10), and (12), respectively. It only remains to verify that, given firms' strategies, optimal buyer search is indeed characterized by the reservation price p^*, i.e. that $W(p_1) > \sigma$ if and only if $p_1 > p^*$. This is where issues associated with equilibrium learning will be most important.

Consider first $p_1 < p^*$; then $W(p_1) = V_m(c_1) < V_m(c^*) = \sigma$, since by definition $p^* = p_m(c^*)$. Next, if $p_1 = p^*$, consumers only infer that $c_1 \in [c^*, c^s)$, so,

$$W(p_1)=\int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c^* \le c_1 < c^s)dc_2$$

$$\le \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c_1 = c^*)dc_2 = V_m(c^*) = \sigma ,$$

and they still do not want to search. For out-of-equilibrium prices $p_1 \in (p^*, p^s)$, assume that they lead to the belief that $c_1 = c^*$, so that:

$$W(p_1)=\int_{p^-}^{p_1} D(p_2)G(p_2 \mid p_1)dp_2 = \int_{p^-}^{p^*} D(p_2)G(p_2 \mid p_1)dp_2 + [S(p^*) - S(p_1)]G(p^* \mid p_1)$$

$$= V_m(c^*) + [S(p^*) - S(p_1)]F(c^* \mid c^*) > \sigma ,$$

due to (4) and $c^* > c^-$. More generally, any out-of-equilibrium beliefs which put sufficient weight on c_1 being closer to c^* than to c^s will lead to search at prices $p_1 \in (p^*, p^s)$.[12]

Finally, buyers will search at all $p_1 \ge p^s$ if and only if $W(p_1) > \sigma$; given firms' strategies, this is equivalent to $V(c_1) \ge \sigma$ for all $c_1 \ge c^s$, where:

$$V(c_1) \equiv \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c_1)dc_2 + [S(p^*) - S(p^s)]F(c^s \mid c_1)$$

$$+ \int_{c^s}^{c_1} D(p_F(c_2))p_F'(c_2)F(c_2 \mid c_1)dc_2 . \qquad (13)$$

12. The fact that consumers' beliefs do not remain monotonic in the observed price as it moves off the equilibrium path is admittedly unappealing. But one has to choose between such monotonicity and the reservation price property. Indeed, due to bunching, consumers who observe their reservation price p^* strictly do not want to search; if observing $p^* + \epsilon$ did not lead them to infer a lower c_1, hence a lower c_2 and p_2, they would not want to search, a contradition.

Using the definition (8) of c^{*}, the condition for search becomes:

$$[S(p^{*})-S(p^{s})]F(c^{s}|c_{1}) + \int_{c^{s}}^{c_{1}} D(p_{F}(c_{2}))p_{F}'(c_{2})F(c_{2}|c_{1})dc_{2} >$$

$$\int_{c^{-}}^{c^{*}} D(p_{m}(c_{2}))p_{m}'(c_{2})[F(c_{2}|c^{*})-F(c_{2}|c_{1})]dc_{2}, \qquad \forall c_{1} \geq c^{s}. \qquad (14)$$

The left hand side is the expected return, conditional on c_1, from finding $p_2 \in [p^{*}, p_1]$; denote it as $\xi(c_1, \sigma)$. The right hand side is the "bad news" from inferring c_1 rather than c^{*}, about the likelihood of finding a price $p_2 < p^{*}$; denote it as $\chi(c_1, \sigma)$. For buyers to have a reservation price rule, $\chi(c_1, \sigma)$ must not be too large. From (14) we see that such will be the case in two intuitive cases, which extend the insights of Rosenfield and Shapiro [1981] and Rothschild [1973] to an equilibrium context. The first one is when σ is small, so that c^{*} is close to c^{-}, making the integral in χ small; the following proposition formalizes this intuition.

<u>Proposition 4:</u> If search costs are relatively low, there exists a pure-strategy, reservation-price equilibrium characterized by critical cost levels c^{*} and c^{s}, with $c^{-} < c^{*} < c^{s} < c^{-}$. Buyers have reservation price $p^{*} = p_m(c^{*})$. Firms charge $p_m(c)$ if $c \in [c^{-}, c^{*}]$, p^{*} if $c \in [c^{*}, c^{s})$, and $p_F(c)$ if $c \in [c^{s}, c^{-}]$, with $c \leq p_F(c) \leq p_m(c)$.

<u>Proof:</u> See appendix.

The second case in which a reservation price rule will be optimal for buyers is when a firm's cost does not reveal too much information about that of its competitor, i.e., when $F_2(c_2 | c_1)$ is small, so that the two distribution functions in χ are close to one another.

Proposition 5: If firms' costs are not too correlated, i.e., if,

$$\overline{F}_2 \equiv \max\{F_2(c_2 \mid c_1) \mid c^- \leq c_2 \leq c_1 \leq c^+\}$$

is not too large, there exists a pure-strategy reservation price equilibrium. For σ above some level σ^*, it involves no search and corresponds to that of Proposition 2 or 3, depending on whether $V_m(c^+)$ is larger or smaller than σ. For $\sigma < \sigma^*$, it involves search and corresponds to the equilibrium in Proposition 4.

Proof: See appendix.

5. Mixed strategy equilibrium. If the assumptions of low search costs or low correlation do not hold, there may be no reservation price equilibrium, as one should expect. Indeed, simulations suggest that there exists an intermediate range of search costs in which none of the three types of equilibria discussed above exists (see Section IV). We must therefore turn briefly to a fourth type of equilibrium, which involves mixed strategies by consumers and generally does not have the reservation price property.

In such an equilibrium, pricing at low levels of costs remains unchanged, i.e., $p(c) = p_m(c)$, for $c \leq c^*$. From c^* to c^+ the pricing rule $p_R(c)$ makes consumers indifferent between searching and not, and they will randomize this decision. Thus for all $c_1 \in [c^*, c^+]$:

$$\int_{c^-}^{c^*} [S(p_m(c_2)) - S(p_R(c_1))] f(c_2 \mid c_1) dc_2 + \int_{c^*}^{c_1} [S(p_R(c_2)) - S(p_R(c_1))] f(c_2 \mid c_1) dc_2 = \sigma. \quad (15)$$

Differentiating this expression with respect to c_1 yields $p_R'(c_1)$ as a function of all $p_R(c_2)$, for $c_2 < c_1$; this allows the function $p_R(.)$ to be constructed, moving up from the

initial condition $p_R(c^*) = p^*$. The fraction $\omega_R(p)$ of consumers who search at any price $p > p^*$ must then be such that it makes the pricing rule $p_R(c)$ optimal; for all $c_1 \in (c^*, c^+]$:

$$p_R(c_1) \in argmax\left\{ \Pi(p, c_1)\left[1 - \omega_R(p) + \int_{p_R^{-1}(p)}^{c^*} \omega_R(p_R(c_2)) f(c_2 \mid c_1) dc_2 \right], p \geq p^* \right\} \tag{16}$$

Note that in this type of equilibrium $p_R(c^+) > c^+$. The highest-cost firm makes positive profits because not all its consumers search; yet if it raised its price they all would search, and it would then make zero profits.

Such an equilibrium is somewhat less appealing intuitively than the previous reservation price equilibria. It is also much more difficult to construct for a general specification: one must show the existence of solutions $p_R(c)$ to (15) and $\omega_R(p) \in [0, 1]$ to (16), both of which are extremely complicated. We have not established general conditions under which this equilibrium exists, but report below on simulations using simple functional forms which indicate that it does exists in the intermediate range where none of the other three equilibria do.

IV. The Effects of Inflation Uncertainty

We are now ready to analyze how equilibrium pricing and search rules are affected by changes in the underlying distribution of costs. We are interested in particular in how an increase in inflation uncertainty impacts firms' market power, the amount of search, profits and consumer surplus. Again, what we label inflation uncertainty is strictly speaking aggregate cost uncertainty. The reader who does not subscribe to our interpretation can thus safely view it as arising from real shocks (raw materials, weather). Hopefully, once we have detailed its interactions with price information and market efficiency, she will share some of our conviction that nominal inflation gives rise to similar issues and effects.

First we demonstrate the two most important intuitions, regarding what we call the correlation effect and the variance effect. This is done by analyzing the comparative statics of c^*, using a particularly convenient specification where costs are jointly log-linear and demand is iso-elastic. Recall that c^* characterizes consumer's reservation price $p^* = p_m(c^*)$, and is also the cost at which firms are first constrained by search from charging their monopoly price. As explained below, c^* provide a partial but generally good indicator of monopoly power in the model; this is confirmed by a variety of simulations using an alternative specification (costs are the sum of uniform private and joint cost shocks and demand is linear), which we report in the second part of this section.

In the no-search equilibrium of Proposition 3, it is clear that an increase in c^* to $c^{*\prime}$ results in prices which are equal up to c^* and greater above. The effects of an increase in c^* on the search equilibrium of Proposition 4 are more complicated. Since it implies an increase in p^*, prices are again equal from c^- to c^* and greater from c^* to the lower of c^s and $c^{s\prime}$. However, the effect on c^s of a change in the cost distribution which causes an increase in c^* is ambiguous. We know that if the differential equation (10) which determines prices

$p_F(c)$ above c^s remains unchanged, c^s will also be greater (Lemma 3). This will imply lower prices from c^s to $c^{s\prime}$ since p^* is less than $p_F(c)$.[13] But of course the differential equation (10) will generally be affected in a complicated manner by changes in the inflation process. Thus by no means do the comparative statics of c^* tell the entire story. Nonetheless our simulations show that they do provide robust intuitions about the full equilibrium effects of inflationary uncertainty in this model.

Case 1: Log-Normal Costs, Iso-Elastic Demand

We begin the analysis by making the following distributional assumptions about firms' costs, now denoted C_i, i = 1,2:

(i) $C_i = \Theta\Gamma_i$; $c_i \equiv \log C_i$

(ii) $\log\Theta \equiv \theta$ is distributed normally with mean 0 and variance υ_θ;

(iii) $\log\Gamma_i \equiv \gamma_i$ is distributed normally with mean $\bar{c}$ and variance υ_γ;

(iv) γ_1, γ_2, and θ are independent.[14]

The distribution of c_2 conditional on c_1 is therefore normal, with mean $\rho c_1 + (1-\rho)\bar{c}$ and variance $(1-\rho^2)(\upsilon_\theta + \upsilon_\gamma)$, where $\rho \equiv \frac{\upsilon_\theta}{\upsilon_\theta + \upsilon_\gamma}$ is the correlation coefficient of the two log-costs c_1 and c_2. We shall examine how υ_θ, which measures aggregate cost or inflation uncertainty, affects the conditional distribution $F(c_1 \mid c_2)$ and consumers' return to search.

13. Note, however, that a firm earns greater profits at this lower price since it does not induce search, while it could have charged $p_F(c)$ and permitted search.

14. Note that this example does not satisfy the assumption of the model that $c^* < +\infty$. This makes it difficult to prove the existence of an equilibrium with search, largely because we can not use $p_F(c^*) = c^*$ as a terminal condition for (10). However, this formulation provides the clearest way to analyze the effects of inflation uncertainty on c^*, because closed-form expressions can be obtained.

Assume that demand is iso-elastic, $D(P) = P^{-\eta}$, $\eta > 1$, so that the log of the monopoly price is: $p_m(c) \equiv \log P_m(c) = \log\left(\frac{\eta}{\eta-1}\right) + c$. Define:

$$\bar{p} \equiv \log\left(\frac{\eta}{\eta-1}\right) + \bar{c}; \qquad \mu(p) \equiv \rho p + (1-\rho)\bar{p}; \qquad s^2 \equiv (1-\rho^2)(\upsilon_\theta + \upsilon_\gamma); \quad (17).$$

The unconditional distribution of $p_m(c)$ is normal, with mean $\bar{p}$ and variance υ_θ. The distribution of $p_m(c_2)$, conditional on $c_1 = c$, is normal with mean $\mu(p_m(c))$ and variance s^2. To better demonstrate the two effects of inflation uncertainty, let us for the moment consider ρ and s^2, rather than υ_θ and υ_γ, as the parameters of interest. We can write consumer surplus at the monopoly price as:

$$S(p_m(c)) = \frac{1}{\eta-1} P_m(c)^{1-\eta} = \frac{1}{\eta-1}\exp[(1-\eta)p_m(c)].$$

Therefore the return to search in the region of monopoly prices is:

$$V_m(c) = \frac{1}{\eta-1}\int_{-\infty}^{p_m(c)} \frac{1}{\sqrt{2\pi}s}\{\exp[(1-\eta)p_2] - \exp[(1-\eta)p_m(c)]\}\exp\left[-\frac{(p_2-\mu)^2}{2s^2}\right]dp_2.$$

with $\mu = \mu(p_m(c))$. Rewriting in terms of the distribution Φ and density ϕ of a standard normal:

$$V_m(c) = \frac{1}{\eta-1}\exp\left[-\frac{1}{2}(\eta-1)[2\bar{p} + 2\rho(p_m(c)-\bar{p}) - s^2(\eta-1)]\right]\cdot\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p}) + s^2(\eta-1)}{s}\right]$$

$$-\frac{1}{\eta-1}\exp[-(\eta-1)p_m(c)]\cdot\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})}{s}\right]. \qquad (18)$$

Hence:

$$\frac{\partial V_m(c)}{\partial\rho} = -(p_m(c)-\bar{p})\cdot\exp-\frac{1}{2}(\eta-1)[2\bar{p} + 2\rho(p_m(c)-\bar{p}) - s^2(\eta-1)]$$

$$\cdot\,\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p}) + s^2(\eta-1)}{s}\right].$$

$$\frac{\partial V_m(c)}{\partial s} = \phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})+s^2(\eta-1)}{s}\right] + s(\eta-1)\Phi\left[\frac{(1-\rho)(p_m(c)-\bar{p})+s^2(\eta-1)}{s}\right]$$
$$\cdot \exp -\frac{1}{2}(\eta-1)[2\bar{p}+2\rho(p_m(c)-\bar{p})-s^2(\eta-1)].$$

So finally:

$$sgn\frac{\partial V_m(c)}{\partial \rho} = sgn(\bar{p}-p_m(c)) = sgn(\bar{c}-c) \tag{19}$$

$$\frac{\partial V_m(c)}{\partial s} > 0. \tag{20}$$

The first result (19) shows what we call the correlation effect. Recall that the mean of the distribution of c_2 conditional on $c_1 = c$ is a weighted average of the observation c and the unconditional mean $\bar{c}$, with weights ρ and $1-\rho$, respectively. Therefore, the mean of this conditional distribution is increasing in ρ if $c > \bar{c}$, and decreasing if $c < \bar{c}$. By increasing ρ, the first effect of an increase in v_θ is therefore to raise the value of search at $c < \bar{c}$, and to lower it at $c > \bar{c}$.[15] This correlation effect captures the idea that inflation or aggregate cost uncertainty makes people <u>search less when they see a high price</u>, because they think that things are just as bad elsewhere. However, it also makes them <u>search more when they see a low price</u>, because they think that there are other, possibly better bargains to be found.

The second result (20) shows what we call the variance effect. Given that buyers can return to the first store costlessly, an increase in the variance s^2 of the conditional distribution

15. Of course, what really determines search is not the conditional distribution of cost c_2, but the conditional distribution of surplus $S(p_2)$; see (3). The difference between the two involves the equilibrium pricing rule $p(c)$ as well as the convexity of the surplus function $S(p)$. This discussion, and the one which follows, are meant to give the main qualitative intuitions.

increases the option value of search. But note that an increase in v_θ, the unconditional variance of the joint cost shock, does lead to such an increase in the conditional variance:

$$\frac{\partial s^2}{\partial v_\theta} = \frac{\partial}{\partial v_\theta}\left[1-\left(\frac{v_\theta}{v_\theta + v_\gamma}\right)^2\right](v_\theta + v_\gamma) = \frac{v_\theta^2}{(v_\theta + v_\gamma)^2} > 0 .$$

thereby raising the value of search and reducing the market power of firms.

We now examine how these two effects impact consumers' reservation price and firms' pricing. We focus on the case where search matters, i.e. where $c^* \equiv \inf\{c \mid V_m(c) = \sigma\} < \infty$. Then $V_m'(c^*) > 0$, so (19) and (20) imply:

$$sgn\frac{\partial c^*}{\partial \rho} = sgn[p_m(c^*) - \bar{p}] = sgn(c^* - \bar{c}) \tag{21}$$

$$\frac{\partial c^*}{\partial s} < 0 . \tag{22}$$

If c^* exceeds $\bar{c}$, the correlation effect tends to increase it further, resulting in monopoly markups over a wider range of costs; the variance effect, however, works in the opposite direction. By definition, such a configuration with $c^* > \bar{c}$ occurs when search is relatively costly. With a relatively low cost of search, on the other hand, c^* is less than $\bar{c}$; in this case both the variance and the correlation effects reduce it even more, and the market becomes more competitive.

While this log-normal case is very specific, the intuitions behind the correlation and variance effects seem quite robust. One can think in general of the variability of inflation, or of any common shock to firms' costs, as having two effects on the conditional distribution of costs (and through equilibrium pricing, of surplus). First, by making costs more correlated, it shifts $F(c_2 \mid c_1)$ and its conditional mean, up for high c_1, down for low c_1; secondly, since it is a source of additional uncertainty, it causes a mean-preserving spread in the shifted

distribution. The generality of these intuitions is also supported by the simulations, using a very different specification, reported below.

In addition to these two central, information-related effects, inflationary uncertainty has other consequences in our model. First, even in the absence of search, an increase in the variance of prices can be beneficial to consumers, because consumer surplus is convex in price. This is clearly a result of our partial equilibrium framework, where the marginal utility of income is constant; [16] we thus tend to de-emphasize this effect, but one should be aware that it is present (for instance in the simulations). Similarly, equilibrium profits may be positively affected because they depend in part on monopoly profits, which are convex in cost. Note however that firms face both ex-ante and ex-post uncertainty, because they set their price after learning their cost, but before learning that of their competitor. The third additional effect is more robust and interesting. Even though an increase in the <u>value</u> of search may raise each firm's elasticity of demand, an increase in search <u>activity</u> shifts consumers and purchases toward the firm with the lower cost. Since these firms are more profitable, this tends to increase expected profits and efficiency at the same time.

---

16. Also, this effect has little to do with information, since it would remain even if search was costless and consumers always bought at the cheapest price.

Case 2: Uniform Costs, Linear Demand

Let us now assume that costs are the sum of a joint cost shock and a private cost shock, and that demand is linear:

(i) $c_i = \bar{c} + \theta + \gamma_i$;

(ii) θ is uniformly distributed on $[-a, a]$;

(iii) γ_i is uniformly distributed on $[-b, b]$;

(iv) γ_1, γ_2, and θ are independent;

(v) $D(p) = A - p$.

We assume that $a < b$, which reduces the number of cases to analyze but is not essential; a represents the volatility of inflation. The unconditional density of cost, $h(c)$ is the sum of two uniform distributions and has the familiar trapeze shape:

$$h(c) = \frac{c - \bar{c} + a + b}{4ab} \qquad if \quad \bar{c} - a - b \le c \le \bar{c} + a - b$$

$$h(c) = \frac{1}{2b} \qquad if \quad \bar{c} + a - b \le c < \bar{c} - a + b$$

$$h(c) = \frac{\bar{c} + a + b - c}{4ab} \qquad if \quad \bar{c} - a + b \le c \le \bar{c} + a + b .$$

The distribution of c_2 conditional on c_1 is also the sum of two uniforms; inferring c_1 from a price observation causes a consumer to update his beliefs about the joint cost shock θ as follows:

if $c_1 \in [\bar{c} - a - b, \bar{c} + a - b]$ the posterior of θ is $\theta \sim U[-a, c_1 - \bar{c} + b]$;

if $c_1 \in [\bar{c} + a - b, \bar{c} - a + b]$ the posterior of θ is $\theta \sim U[-a, a]$;

if $c_1 \in [\bar{c} - a + b, \bar{c} + a + b]$ the posterior of θ is $\theta \sim U[c_1 - \bar{c} - b, a]$.

Note that if c_1 falls in the intermediate region there is no learning. However, if c_1 falls in the lowest region, the conditional expectation of c_2 is less than $\bar{c}$, and decreasing in the variability α, while if c_1 falls in the highest region, the conditional expectation of c_2 is above $\bar{c}$, and increasing in α. Thus the correlation effect works here in a way similar to the log-normal case. The same is true of the variance effect, since the support of θ, given c_1, always widens as α increases.

We now look at a number of simulations of this example, in order to get some feeling for the relative size of the various effects of an increase in aggregate uncertainty. In all simulations, $D(p) = 15 - p$, $\bar{c} = 6$, and $b = 3$. We allow search costs σ and the dispersion of the joint cost shock α to vary. The results for low, intermediate, and high search costs are given in Tables 1, 2, and 3, respectively. We define these terms so that low search costs lead to a c^* well below the unconditional mean of $\bar{c} = 6$, intermediate search costs lead to a c^* near $\bar{c}$, and high search costs lead to a c^* well above $\bar{c}$.

Looking first at the effects of search costs, we see that as they increase, the equilibrium first involves reservation price strategies and search, then mixed strategies, then monopolistic pricing plus bunching at p^*, and finally unconstrained monopolistic pricing. These results support the intuitive way in which we associated each type of equilibrium to a different range of search costs.

Next we turn to our main subject of interest: the effects of inflation uncertainty on monopoly power and on the components of welfare in equilibrium.

For low search costs, which are reported in Table 1, c^* is decreasing in α, because both the variance and correlation effects make search more valuable. This reduction in equilibrium pricing strategies with increases in the variance of the joint cost shock leads to gains in consumer surplus.

LOW SEARCH COSTS

$\underline{\sigma = 0.1}$:

a	Type	c^*	c^s	p^s	Π	CS	Welfare	E(c)	E(p)
0.10	2	3.6413	--	--	18.863	16.227	35.090	6.0000	9.3034
0.50	3	3.5719	9.3189	9.3789	18.827	16.425	35.252	5.9919	9.2688
1.00	3	3.0871	8.9957	9.3204	18.814	17.779	36.593	5.8663	9.0151
1.50	3	2.5647	8.6442	9.2282	18.834	19.270	38.104	5.6789	8.6989
2.00	3	2.0438	8.2318	9.0783	18.899	20.692	39.591	5.4329	8.3351
2.50	3	1.5240	7.7904	8.8921	18.995	21.966	40.961	5.1407	7.9309

$\underline{\sigma = 0.5}$:

a	Type	c^*	c^s	p^s	Π	CS	Welfare	E(c)	E(p)
0.10	2	4.4747	--	--	19.791	14.248	34.039	6.0000	9.6466
0.50	2	4.4435	--	--	19.691	14.430	34.121	6.0000	9.6315
1.00	3	4.3420	9.6651	9.7756	19.687	14.697	34.384	5.9860	9.5816
1.50	3	3.9877	9.4448	9.7844	19.745	15.550	35.295	5.9004	9.4066
2.00	3	3.4317	9.0605	9.6659	19.793	16.923	36.716	5.7316	9.0955
2.50	3	2.8796	8.6373	9.5031	19.880	18.281	38.161	5.5029	8.7325

Table 1

NOTES:
In all simulations $D(p) = 15\text{-}p$, $\bar{c} = 6$, $b = 3$.

Type refers to the equilibrium pricing strategy in the following way:

1 = $p(c) = p_m(c)\ \ \forall c$. No search.
2 = $p(c) = p_m(c)\ \ \forall c \le c^*$, $p(c) = p^*\ \ \forall c > c^*$. No search.
3 = $p(c) = p_m(c)\ \ \forall c < c^*$, $p(c) = p^*\ \ \forall c \in [c^*, c^s)$, $p(c) = p_F(c)\ \ \forall c \ge c^s$. Search above p^*.
4 = $p(c) = p_m(c)\ \ \forall c < c^*$, $p(c) = p_F(c)\ \ \forall c \ge c^*$. Consumers search with a mixed strategy above $\mathrm{Max}\{c^*, \bar{c} - a + b\}$.

Π and CS are, respectively, expected industry profits and consumer surplus per customer (net of search costs).

E(p) and E(c) are, respectively, the expected price that a customer pays and the expected cost at the store she purchases from.

INTERMEDIATE SEARCH COSTS

$\sigma = 2.5$:

a	Type	c^*	c^s	p^s	Π	CS	Welfare	E(c)	E(p)
0.10	2	6.5259	--	--	20.790	11.476	32.266	6.0000	10.2448
0.50	2	6.5098	--	--	20.798	11.510	32.308	6.0000	10.2381
1.00	2	6.4592	--	--	20.820	11.782	32.602	6.0000	10.2171
1.50	2	6.3740	--	--	20.854	11.801	32.655	6.0000	10.1814
2.00	4	6.2524	--	--	21.358	11.581	32.939	5.9482	10.2333
2.50	4	6.0917	--	--	21.584	11.573	33.157	5.9431	10.2504

$\sigma = 3.0$:

a	Type	c^*	c^s	p^s	Π	CS	Welfare	E(c)	E(p)
0.10	2	6.9156	--	--	20.854	11.170	32.024	6.0000	10.319
0.50	2	6.9004	--	--	20.886	11.200	32.086	6.0000	10.313
1.00	2	6.8528	--	--	20.916	11.297	32.213	6.0000	10.294
1.50	2	6.7727	--	--	20.964	11.461	32.425	6.0000	10.262
2.00	4	6.6590	--	--	21.396	11.165	32.561	5.9645	10.342
2.50	4	6.5462	--	--	21.588	11.074	32.662	5.9695	10.389

Table 2

NOTES:
In all simulations D(p) = 15-p, $\bar{c} = 6$, b = 3.

Type refers to the equilibrium pricing strategy in the following way:

1 = $p(c) = p_m(c) \quad \forall c$. No search.
2 = $p(c) = p_m(c) \quad \forall c \leq c^*$. $p(c) = p^* \quad \forall c > c^*$. No search.
3 = $p(c) = p_m(c) \quad \forall c < c^*$. $p(c) = p^* \quad \forall c \in [c^*, c')$. $p(c) = p_f(c) \quad \forall c \geq c'$. Search above p^*.
4 = $p(c) = p_m(c) \quad \forall c < c^*$, $p(c) = p_f(c) \quad \forall c \geq c^*$. Consumers search with a mixed strategy above $\mathrm{Max}\{c^*, \bar{c} - a + b\}$.

Π and CS are, respectively, expected industry profits and consumer surplus per customer (net of search costs).

E(p) and E(c) are, respectively, the expected price that a customer pays and the expected cost at the store she purchases from.

HIGH SEARCH COSTS

$\sigma = 4.0$:

a	Type	c^*	c^s	p^s	Π	CS	Welfare	E(c)	E(p)
0.10	2	7.6430	--	--	20.966	10.766	31.732	6.0000	10.423
0.50	2	7.6289	--	--	20.980	10.792	31.772	6.0000	10.418
1.00	2	7.5849	--	--	21.024	10.873	31.897	6.0000	10.403
1.50	2	7.5435	--	--	21.100	10.996	32.096	6.0000	10.380
2.00	2	8.7607	--	--	21.312	10.777	32.089	6.0000	10.461
2.50	2	10.448	--	--	21.520	10.767	32.287	6.0000	10.497

$\sigma = 5.0$:

a	Type	c^*	c^s	p^s	Π	CS	Welfare	E(c)	E(p)
0.10	2	8.3300	--	--	20.986	10.561	31.547	6.0000	10.481
0.50	2	8.3165	--	--	21.014	10.584	31.598	6.0000	10.477
1.00	2	9.1327	--	--	21.082	10.554	31.636	6.0000	10.496
1.50	1	--	--	--	21.188	10.594	31.782	6.0000	10.500
2.00	1	--	--	--	21.333	10.667	32.000	6.0000	10.500
2.50	1	--	--	--	21.520	10.760	32.280	6.0000	10.500

Table 3

NOTES:
In all simulations D(p) = 15-p, $\bar{c} = 6$, b = 3.

Type refers to the equilibrium pricing strategy in the following way:

1 = $p(c) = p_m(c)$ $\forall c$. No search.
2 = $p(c) = p_m(c)$ $\forall c \leq c^*$, $p(c) = p^*$ $\forall c > c^*$. No search.
3 = $p(c) = p_m(c)$ $\forall c < c^*$, $p(c) = p^*$ $\forall c \in [c^*, c^s)$, $p(c) = p_F(c)$ $\forall c \geq c^s$. Search above p^*.
4 = $p(c) = p_m(c)$ $\forall c < c^*$, $p(c) = p_g(c)$ $\forall c \geq c^*$. Consumers search with a mixed strategy above $\mathrm{Max}\{c^*, \bar{c} - a + b\}$.

Π and CS are, respectively, expected industry profits and consumer surplus per customer (net of search costs).

E(p) and E(c) are, respectively, the expected price that a customer pays and the expected cost at the store she purchases from.

Another interesting aspect of these simulations is that profits do not decrease much with increases in α. In fact, for high levels of α profits are actually increasing in α. There are two reasons for this. First as α increases, search increases. This raises the likelihood that consumers will purchase at a low-cost firm, which has higher profits per customer. Thus, the better matching of consumers to low-cost firms actually tends to increase profits. One can see this in the last two columns of Table 1, which report the price paid by the average consumer and the cost incurred for the average customer. At higher levels of α, the decrease in price is offset by the decrease in cost. The second factor which offsets the deleterious effect of lower prices on producers is the fact that monopoly profits are convex in cost, and thus increase in expectation with the unconditional variability α .

Table 2 reports simulation results for intermediate values of σ. When c^* is near $\bar{c}$, the correlation effect affects it very little or not at all. The variance effect alone acts to reduce c^* as α increases, but this reduction is very limited compared to Table 1. Similarly, the total effect of inflation variability on consumer surplus and profits is small relative to the previous case. These simulations also indicate that the determination of c^* is not the entire story, because in some ranges c^* is decreasing, yet profits increase and consumer surplus decreases.

Table 3 reports the results when c^* is significantly greater than $\bar{c}$. In all equilibria, there is no search. This is partially a result of the linear demand, uniform cost specification. When $\sigma = 4$, c^* increases when α is sufficiently large; the correlation effect then becomes dominant. In this range, consumer surplus decreases, but by less than profits increase. The increased variance effect mitigates some of the harm of the correlation effect.

Indeed, one noteworthy feature of all the simulations is that social welfare is always increasing or almost constant in the variance of inflation. In those cases where consumer are adversely affected, producers are made better off by at least an equal magnitude.

The lesson to take away from these simulations, however, is not that stochastic inflation is welfare-improving. This result clearly depends a great deal on the exact specification of the model. Rather, we interpret them as saying that one needs to know a great deal more, in particular about the size of informational costs, before one can say that increases in stochastic inflation reduce the informativeness of prices and thereby decrease welfare. In an economy where endogenous information gathering feeds back into equilibrium price setting, several complex effects are at play, and one cannot say a priori what their net impact on welfare will be without simultaneously making a statement about market structure. The simulations show that it is conceivable that the benefits of increases in the variance of joint cost shocks outweigh the losses.

V. Conclusion

Aggregate cost uncertainty, whether due to common input prices or to stochastic inflation, reduces the information content of prices by making it difficult to separate relative and aggregate price variations. In this paper we have explored how this mechanism operates in an environment where agents can act to enhance their information via search. We studied how the stochastic structure of shocks, consumer search, and non-competitive pricing interact in a single product market.

The results indicate that the a priori case for significant welfare losses from inflation associated with reduced informativeness of prices becomes much weaker when one allows for endogenous information acquisition and price-setting. Indeed, inflationary noise can lead agents to seek more information, so that in equilibrium they will in fact be better informed, and prices will reflect increased competition. The decisive factor in whether inflation uncertainty improves or deteriorates market efficiency will be the size of informational costs.

Another contribution of this paper to the theoretical literature is that it develops an equilibrium model in which consumers search optimally from an unknown price distribution, and firms price optimally given the learning and search rules of consumers. We hope that this analysis will be useful for attaining a better understanding of the relation between pricing and search behavior in general.

There remains of course much to be done before we can fully assess the possibility of information-related welfare losses from stochastic inflation. Our model has a very specific market structure and information technology, in which the effect of noisy inflation on the incentive to search and on pricing may very well offset the reduced informativeness of the cost structure. It would be useful to extend the analysis to other market structures and information technologies, so as to ascertain the robustness of our results. In particular, making the model

dynamic by incorporating repeat purchases seems quite desirable, although quite difficult. Finally, one should also introduce general equilibrium effects, in order to make the inflationary interpretation more precise.

Appendix: Proofs of Propositions

<u>Proof of Proposition 1:</u> Let the lowest price charged in a given equilibrium of the game be p^-. First, assume $p^- > p_m(c^-)$. If firm i has cost c^- then if it charges $p_m(c^-)$, it gets at least as many customers as at any other price. This is because a consumer who observes $p_m(c^-)$ will not search since there is zero probability of finding a lower price. In addition, all consumers who first go to the other store and decide to search will purchase from firm i, since it has the lowest price charged in equilibrium. Since firm i's profits per customer are maximized at $p_m(c^-)$, the deviation is profitable. Thus $p^- \leq p_m(c^-)$.

Assume $p^- < p_m(c^-)$. Let us implicitly define ϵ_1 by $S(p^-) - \sigma + \delta = S(p^- + \epsilon_1)$, where δ is an arbitrarily small positive number. No consumer will wish to search if he observes $p^- + \epsilon_1$ since even if he were sure to find the lowest price possible, the gain is insufficient to cover the search costs. Given the quasi-concavity of profits, a firm with cost c^- earns more per customer at $p^- + \epsilon_1$ than at p^-. Furthermore, the firm will get as many customers by deviating. Therefore, the deviation is profitable and $p^- = p_m(c^-)$.

Since no consumers search at prices sufficiently close to p^- a firm with cost $c^- + \epsilon$ where ϵ is a small positive number, will charge its monopoly price, since it maximizes the number of customers as well as the profits per customer. Q.E.D.

<u>Proof of Proposition 2:</u> A consumer's search decision for any price played in equilibrium is to purchase from the first store and not search. Given the equilibrium pricing strategy if he observes p_i at the first store, the value to search is $V_m(c_i) \leq \max_{[c^-, c^+]} V_m(c) < \sigma$. Thus, it does not pay to search. Given that no consumer searches in equilibrium, the maximum number of customers the firm can attract is half of all customers. Since the number of buyers is not sensitive to price, (except if a firm charges a price greater than $p_m(c^+)$ the number of customers could be lower) a firm charges a price which maximizes profit per customer which is $p_m(c)$. This is the optimal price to charge independent of off-the-equilibrium-path behavior of consumers. Q.E.D.

<u>Proof of Proposition 3:</u> In this equilibrium, consumers' search rule is to search if and only if the first observed price exceeds p^*. By the definition of c^*, no consumer wishes to search at prices below p^*. At p^*, all a consumer knows is that $c_1 \in [c^*, c^+]$. By the definition of c^*, he is indifferent between searching and not if he

observes p^* and knows that $c_1 = c^*$. However, in this equilibrium, observing p^* only reveals that $c_1 \geq c^*$. Given the positive correlation in costs this implies that the consumer's beliefs of c_2 are at least as great as if he knew $c_1 = c^*$. This combined with the (weak) monotonicity of the pricing rule implies that he does not wish to search if he observes p^* in the proposed equilibrium. This shows that consumer search decisions are optimal on the equilibrium path, where all prices are below $p_m(c^-)$. If a consumer observes a price above p^*, his beliefs must be such that it pays for him to search. Believing that $c_1 = c^*$, or more generally that c_1 is close to c^* is sufficient to ensure that he does wish to search.

Pricing rules are clearly optimal. If any firm deviates to a price greater than p^* it gets zero consumers, since they search and find a lower price. Thus, so long as a firm charges a price no greater than p^*, it gets half the consumers. Thus, the optimal price is $p_m(c)$ unless it exceeds p^* in which case (given the quasi-concavity of profits) the optimal price is p^*. Q.E.D.

<u>Proof of Lemma 1:</u> Since the differential equation:

$$p'(c) \quad = \quad \frac{f(c\,|\,c)}{1-F(c\,|\,c)} \cdot \frac{\Pi(p(c),c)}{\Pi_p(p(c),c)} \qquad (A.1)$$

does not satisfy Lipschitz conditions at $(p(c),c) = (c^*,c^*)$ nor at any $(p_m(c),c)$, standard theorems are not applicable. Instead, $p_F(c)$ will be constructed as the fixed point of a contraction mapping. Let C_0 be the space of continuous functions on $[0,c^*]$, and:

$$C \quad \equiv \quad \{p(\cdot) \in C_0 \,|\, p(c) \in [c, p_m(c)] \quad \forall c\}. \qquad (A.2)$$

Denote by $\nu(c) \equiv \frac{f(c|c)}{1-F(c|c)}$ the hazard rate entering (A.1). $p(\cdot)$ solves (A.1) if and only if the function

$$I(c) \equiv \Pi(p(c),c) \qquad (A.3)$$

obeys the differential equation:

$$I'(c) \quad = \quad \Pi_p(p(c),c)p'(c) - D(p(c)) \quad = \quad \nu(c)I(c) - D(p(c)) \qquad (A.4)$$

with terminal condition $I(c^*) = \Pi(c^*,c^*) = 0$. Integrating (A.4) backwards:

$$I(c) \quad = \quad \int_c^{c^*} D(p(x))e^{-\int_c^x \nu(y)dy} dx \quad \equiv \quad J(p(\cdot),c). \qquad (A.5)$$

This integral is convergent at c^*, since $p(x) \geq x$ implies:

$$I(c) \quad \leq \quad \int_c^{c^*} D(x)dx \quad = \quad S(c) - S(c^*).$$

We have transformed the differential equation (A.1) into an equivalent integral equation:

$$J(p(\cdot),c) = \Pi(p(c),c), \quad \forall c. \qquad (A.6)$$

By assumption $\Pi(\cdot,c)$ is strictly quasi-concave, hence can be inverted from $[0,\Pi_m(c)]$ into $[c,p_m(c)]$. Hence, with obvious notation, the fixed point formulation (clearly $I(c)\in[0,\Pi_m(c)]$) is:

$$p(c) = \Pi(\cdot,c)^{-1}(J(p(\cdot),c)). \qquad (A.7)$$

We now show that the mapping $T:p(\cdot)\to Tp(\cdot)$, where $Tp(c)$ is the r.h.s. of (A.7), is a contraction on C endowed with the sup norm: $\|p\| = \sup_{c\in[0,c^*]} |p(c)|$. Let $p(\cdot)\in C$; by construction, $Tp(c)\in[c,p_m(c)]$, $\forall c$.

Moreover, it is easily verified that $\Pi(\cdot,c)^{-1}(\bar{\Pi})$ is jointly continuous in $(c,\bar{\Pi})$, for all $c\in[c^-,c^*]$ and for all $\bar{\Pi}\in[0,\Pi_m(c)]$. Since $J(p(\cdot),c)=I(c)$ is clearly continuous in c, Tp is then continuous, hence $Tp(\cdot)\in C$. Consider now $(p,q)\in C\times C$ and any $c\in[c^-,c^*]$. We have:

$$|Tp(c)-Tq(c)| = |\Pi(\cdot,c)^{-1}(J(p(\cdot),c)-\Pi(\cdot,c)^{-1}(J(q(\cdot),c))|. \qquad (A.8)$$

Note that $\Pi(\cdot,c)^{-1}$ has derivative $1/\Pi_p(\Pi(\cdot,c)^{-1},c)$, which is unbounded. For all $X,Y\in[0,\Pi_m(c)]$, with $X\geq Y$ denote $x=\Pi(\cdot,c)^{-1}(X)$, $y=\Pi(\cdot,c)^{-1}(Y)$. We claim:

$$\frac{1}{M}\cdot\frac{X-Y}{p_m(c)-y} \leq x-y \leq \frac{1}{m}\cdot\frac{X-Y}{p_m(c)-x}. \qquad (A.9)$$

Indeed there exists Z, $X\geq Z\geq Y$, or $z=\Pi(\cdot,c)^{-1}(Z)$, $x\geq z\geq y$ such that $\frac{X-Y}{x-y}=\Pi_p(z,c)$. But, by (4), the definition of m and M:

$$m\leq\frac{\Pi_p(z,c)}{p_m(c)-z}\leq M.$$

Inequality (A.9) then follows from $y\leq z\leq x\leq p_m(c)$. Next, apply the first inequality in (A.9) with $X'=\Pi_m(c)$, $x'=p_m(c)$, $Y'=X$, $y'=x$:

$$[p_m(c)-x]^2\geq\frac{\Pi_m(c)-X}{M} \quad \text{or} \quad p_m(c)-x\geq\sqrt{\frac{\Pi_m(c)-X}{M}} \qquad (A.10)$$

Finally, replace (A.10) in the second part of (A.9), to obtain:

$$\Pi(\cdot,c)^{-1}(X)-\Pi(\cdot,c)^{-1}(Y)=x-y\leq\frac{\sqrt{M}}{m}\frac{X-Y}{\sqrt{\Pi_m(c)-X}}. \qquad (A.11)$$

Therefore,

$$|Tp(c)-Tq(c)| \le \frac{\sqrt{M}}{m} \cdot \frac{\left| \int_c^{c^*} e^{-\int_c^x v(y)dy} [D(p(x))-D(q(x))]dx \right|}{\sqrt{\Pi_m(c)-J(p(\cdot),c)}}$$

$$\le \frac{\sqrt{M}}{m} \cdot \Delta \frac{\int_c^{c^*} e^{-\int_c^x v(y)dy} |p(x)-q(x)|dx}{\sqrt{\Pi_m(c)-J(p(\cdot),c)}}$$

$$\le |p-q| \cdot \frac{\Delta\sqrt{M}}{m} \cdot \frac{c^*-c}{\sqrt{\Pi_m(c)-J(p(\cdot),c)}}. \tag{A.12}$$

Thus T will be a contraction if, for all $c \in [0, c^*]$,

$$\Pi_m(c)-J(p(\cdot),c) > M\left(\frac{\Delta}{m}\right)^2 (c^*-c^-)^2. \tag{A.13}$$

Indeed, uniform continuity will then imply that (A.13) holds with M replaced by $M/\sqrt{\beta}$, for some $\beta \in (0,1)$, so that $|Tp-Tq| < \beta |p-q|$. But note that we have,

$$\Pi_m(c)-J(p(\cdot),c) = \Pi_m(c) - \int_c^{c^*} e^{-\int_c^x v(y)dy} D(p(x))dx > \Pi_m(c) - \int_c^{c^*} D(x)dx = \Pi_m(c) - S(c) + S(c^*),$$

since $p(c) > c$. Because, $\Pi_m(c)-S(c)$ is increasing, (A.13) then holds by assumption (6). This concludes the proof of the existence and uniqueness of $p_F(c)$.

It remains to show that $p_F'(c) > 0$, $\forall c \in [c^-, c^*]$. For $c \in [c^-, c^*)$, $\Pi(p_F(c), c) > 0$ by (A.5) while $f(c \mid c) > 0$ by assumption (4), hence the result, by (A.1).

The case of the left derivative $p_F'(c^*)$ is more complicated because both the numerator and denominator in (A.1) go to zero as c goes to c^*. For all $\epsilon > 0$,

$$f(c^*-\epsilon \mid c^*-\epsilon)\Pi(p_F(c^*-\epsilon), c^*-\epsilon) - [1-F(c^*-\epsilon \mid c^*-\epsilon)]\Pi_p(p_F(c^*-\epsilon), c^*-\epsilon)p_F'(c^*-\epsilon) = 0. \tag{A.14}$$

But,

$$F(c^*-\epsilon \mid c^*-\epsilon) - F(c^* \mid c^*) = [f(c^* \mid c^*) + F_2(c^* \mid c^*)](-\epsilon) + o(\epsilon),$$

$$\Pi(p_F(c^*), c^*) - \Pi(p_F(c^*-\epsilon), c^*-\epsilon) = [\Pi_p(p_F(c^*-\epsilon), c^*-\epsilon)p_F'(c^*-\epsilon) + \Pi_c(p_F(c^*-\epsilon), c^*-\epsilon)]\epsilon + o(\epsilon)$$

$$= [\Pi_p(c^* \mid c^*)p_F'(c^*-\epsilon) + \Pi_c(c^*, c^*)]\epsilon + o(\epsilon).$$

Since $F(c^* \mid c^*) = 1$ and $\Pi(p_F(c^*), c^*) = 0$, (A.14) becomes:

$$\{f(c^* \mid c^*)[-\Pi_p(c^* \mid c^*)p_F'(c^*-\epsilon) + D(c^*)] - [f(c^* \mid c^*) + F_2(c^* \mid c^*)]\Pi_p(c^*, c^*)p_F'(c^*-\epsilon)\}\epsilon = o(\epsilon),$$

or

$$p_F'(c^* - \epsilon)[2f(c^* \mid c^*) + F_2(c^* \mid c^*)]\Pi_p(c^*, c^*) = f(c^* \mid c^*)D(c^*) + o(1)$$

which means that the limit $p_F'(c^*) = \lim_{\epsilon \to 0} p_F'(c^* - \epsilon)$ exists and, since $\Pi_p(c^* \mid c^*) = D(c^*)$,

$$p_F'(c^*) = \frac{f(c^* \mid c^*)}{2f(c^* \mid c^*) + F_2(c^* \mid c^*)} > \frac{1}{2}.$$

This proves the result. Q.E.D.

<u>**Proof of Lemma 2:**</u> Consumers observe $p_1 \geq p^s$; given that they search, they will come back only if the other firm has $p_2 > p_1$. Given that $p_F(c_2) < p^s$ if $c_2 < c^s$ and $p_F(c_2)$ is increasing in c_2 for $c_2 \geq c^s$, this occurs if and only if $p_F(c_2) > p_1$ and has probability $F(p_F^{-1}(p_1) \mid c)$; hence the form of $\Psi(p, c)$. The first-order condition for maximization $\Psi_p(p, c) = 0$ is precisely the differential equation (A.1), to which $p_F(c)$ is the only solution. Thus it only remains to show that $\Psi_{pp}(p_F(c), c) < 0$, implying that equilibrium profits (for $c \geq c^s$) are strictly quasiconcave in p . But by the implicit function theorem,

$$p_F'(c) = \frac{-\Psi_{pc}(p_F(c), c)}{\Psi_{pp}(p_F(c), c)}.$$

Given that $p_F' > 0$, the second-order condition becomes $\Psi_{pc}(p_F(c), c) > 0$. But,

$$\Psi_c(p, c) = -D(p)[1 - F(p_F^{-1}(p) \mid c)] - \Pi(p, c)F_2(p_F^{-1}(p) \mid c).$$

$$\Psi_{pc}(p_F(c), c) = -D'(p_F(c))[1 - F(c \mid c)] + D(p_F(c)) \cdot \frac{f(c \mid c)}{p_F'(c)} - \Pi(p, c) \cdot \frac{f_2(c \mid c)}{p_F'(c)} - \Pi_p(p(c), c)F_2(c \mid c).$$

The first two terms are positive. Replacing $p_F'(c)$ from (A.1), the sum of the last two terms has the sign of:

$$-\Pi_p(p_F(c), c)[f_2(c \mid c)(1 - F(c \mid c)) + F_2(c \mid c)f(c \mid c)] > 0,$$

from assumption (4) and the fact that $\Pi_p(p_F(c), c) > 0$. Q.E.D.

<u>**Proof of Lemma 3:**</u> We first find c^s , the point of indifference between p^* and $p_F(c)$ (when $c^s < c^*$), by examining:

$$\delta(c) = \Pi(p^*, c)\left[1 - \frac{1}{2}F(c \mid c)\right] - \Pi(p_F(c), c)[1 - F(c \mid c)]. \tag{A.15}$$

We first show that if $\delta(c) = 0$, then $\delta'(c) < 0$. Indeed, $\delta(c) = 0$ if and only if,

$$\Pi(p^*, c) = \frac{1 - F(c \mid c)}{1 - \frac{1}{2}F(c \mid c)}\Pi(p_F(c), c) < \Pi(p_F(c), c). \tag{A.16}$$

Then:

$$\delta'(c) = -D(p^*)\left[1-\frac{1}{2}F(c\mid c)\right] - D(p_F(c))[1-F(c\mid c)] - \Pi_p(p_F(c),c)[1-F(c\mid c)]p_F'(c)$$
$$-\left[\frac{1}{2}\Pi(p^*,c) - \Pi(p_F(c),c)\right][f(c\mid c) + F_2(c\mid c)]$$
$$= -D(p^*)\left[1-\frac{1}{2}F(c\mid c)\right]\left[1 - \frac{p^*-c}{p_F(c)-c}\right] - \frac{1}{2}\Pi(p^*,c)f(c\mid c)$$
$$+\left[\Pi(p_F(c),c) - \frac{1}{2}\Pi(p^*,c)\right]F_2(c\mid c),$$

where we used both (A.1) and (A.16). Now (A.16) and (2) imply that the last term is negative, and also that $p^* < p_F(c)$ since $\Pi(\cdot,c)$ is increasing on $[c, p_m(c)]$, which contains $p^*$and $p_F(c)$. This in turn implies that the first term above is also negative. Since the second term is always negative, we have showed that $\delta'(c) < 0$ whenever $\delta(c) = 0$.

Therefore, $\delta(\cdot)$ can have at most one zero. Moreover,

$$\delta(c^*) = \Pi(p^*,c^*)\left[1-\frac{1}{2}F(c^*\mid c^*)\right] - \Pi(p_F(c^*),c^*)[1-F(c^*\mid c^*)] > 0$$

because $p^* = p_m(c^*) > p_F(c^*)$, so $\Pi(p^*,c^*) > \Pi(p_F(c^*),c^*)$, while $\delta(c_*) = \frac{1}{2}\Pi(p^*,c_*) = \frac{1}{2}(p^*-c_*)D(p^*)$ has the sign of p^*-c_*. Thus two cases can arise:

(i) If $p^* \geq c_*$, $\delta(c^*) \geq 0$, so all firms with cost in $[c_*, c^*]$ prefer charging $p^*$to $p_F(c)$ and, therefore, also (by Lemma 2) to any price above p^*. Thus p^* maximizes profits.

(ii) If $p^* < c_*$, $\delta(\cdot)$ has a unique zero $c^s \in (c_*, c^*)$, and a firm with cost above c^s prefers charging $p_F(c)$ to p^*; since $c > c^*$, p^* is its preferred price among those which do not induce search. By Lemma 2, $p_F(c)$ is its preferred price among those which induce search. Thus $p_F(c)$ is the globally optimal price for $c \geq c^s$. For $c \in [c^*, c^s)$, $\delta(c) < 0$ so the firm would rather charge p^* than $p_F(c)$, and also than any $p \leq p^*$, since there is no search below $p^* \leq p_m(c)$ and $\Pi(\cdot,c)$ increases on $[c, p_m(c)]$. Finally, a firm with $c < c^*$ clearly will prefer charging $p_m(c)$. Note that the uniqueness of the solution c^s to $\delta(c) = 0$ ensures that it is continuous in c^*. Moreover, for $c > c^*$, $p^* < p_m(c)$ so $\Pi(p^*,c)$ and $\delta(c)$ increase in p^* or c^*. Therefore, c^s increases in c^*, and so does $p^s = p_F(c^s)$, since $p_F(\cdot)$ is increasing and does not depend on c^*. Q.E.D.

<u>**Proof of Proposition 4:**</u> We shall make the dependence of c^*, c^s, etc., on σ explicit, by denoting them as c^*_σ, c^s_σ, etc.

To show that (14) holds when σ is small enough, we examine more closely the determination of c^* and c^s for small σ. Recall that,

$$V_m(c^*) = \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c^*)dc_2 = \sigma .$$

Even if $V_m(\cdot)$ is not monotonic on all of $[c^-, c^*]$, it is monotonic up to some (maximal) $c^1 \in (c^-, c^*]$, because $V_m'(c^-) > 0$. Moreover, $V_m(c) > 0$ in $[c^1, c^*]$, so $V_m(\cdot)$ is bounded away from zero on this interval, i.e., $V_m(c) \geq \underline{V}_m > 0$. Since $V_m(c^-) = 0$, there exists a unique $c^{**} \in [c^-, c^1]$ such that: $\forall c \in [c^-, c^*]$ $V_m(c) \geq \underline{V}_m$ if and only if $c \geq c^{**}$. Let $\sigma^{**} = V_m(c^{**}) = \underline{V}_m$. Then for all $\sigma \leq \sigma^{**}$, $\exists! c^*_\sigma \in [c^-, c^{**}] \forall c \in [c^- c^*]$, $V_m(c) > \sigma$ if and only if $c > c^*_\sigma$, namely $c^*_\sigma = \max\{c \in [c^-, c^{**}] \mid V_m(c) \leq \sigma\}$. Thus for $\sigma \leq \sigma^{**}$, monopoly pricing can be sustained up to the cost c^*, and not above. Clearly, as σ decreases from σ^{**} to zero, c^*_σ decreases to c^-. In particular, we shall assume that $c^*_\sigma < c_m(c^*)$, i.e, $p^*_\sigma = p_m(c^*_\sigma) < c^*$.

Let us turn next to the determination of c^s_σ. The differential equation (10) and its solution $p_f(\cdot)$ do <u>not</u> depend on σ, while c^s_σ is defined by (see Lemma 3):

$$\Pi(p^*_\sigma, c^s_\sigma)\left[1 - \frac{1}{2}F(c^s_\sigma \mid c^s_\sigma)\right] = \Pi(p_f(c^s_\sigma), c^s_\sigma)[1 - F(c^s_\sigma \mid c^s_\sigma)] .$$

(Recall that $p^*_\sigma < c^*$ ensures that a unique solution $c^s_\sigma \in (c^*_\sigma, c^*)$ exists). As σ decreases to zero, p^*_σ decreases to $p^*_0 = p_m(c^-)$ so by Lemma 3, c^s_σ decreases to a limit $c^s_0 \geq c^-$. In fact c^s_σ remains bounded away from c^-, otherwise in the limit: $\Pi(p_m(c^-), c^-) = \Pi(p_f(c^-), c^-)$, which is impossible since $c^- < p_f(c^-) < p_m(c^-)$ and $\Pi(\cdot, c^-)$ is strictly quasiconcave.

We are now ready to examine (14) for low values of σ. First, for all $c_1 \geq c^s_\sigma$:

$$\chi(c_1, \sigma) \leq \int_{c^-}^{c^*} D(p_m(c_2))p_m'(c_2)F(c_2 \mid c^*)dc_2 = \sigma \tag{A.17}$$

so $\chi(c_1, \sigma)$ goes to zero (uniformly in c_1) with σ. On the other hand we show that:

$$\underline{\xi}(\sigma) = \min\{\xi(\sigma, c_1), c_1 \in [c^s_\sigma, c^*]\} > 0 \tag{A.18}$$

for σ low enough. First note, that for all $\sigma \leq \sigma^{**}$ and $c_1 \geq c^s_\sigma \geq c^s_0$:

$$|\xi(\sigma,c_1)-\xi(0,c_1)| \le |S(p_\sigma^\bullet)-S(p_0^\bullet)| + |S(p_\sigma^s)-S(p_0^s)| + \int_{c_0^s}^{c_\sigma^s} D(p_F(c_2))p_F{}'(c_2)dc_2$$

$$= |S(p_\sigma^\bullet)-S(p_0^\bullet)| + 2|S(p_\sigma^s)-S(p_0^s)|, \qquad (A.19)$$

so $\xi(\sigma,c_1)$ converges to $\xi(0,c_1)$ uniformly in c_1 as σ goes to zero. Therefore, by continuity (A.18) will hold if $\underline{\xi}(0)>0$, i.e., for all $c_1 \in [c_0^s, c^*]$:

$$\xi(0,c_1) = [S(p_0^\bullet)-S(p_F(c_0^s))]F(c_0^s\,|\,c_1) + \int_{c_0^s}^{c_1} D(p_F(c_2))p_F{}'(c_2)F(c_2\,|\,c_1)dc_2 > 0. \quad (A.20)$$

Since $p_0^s = p_F(c_0^s) > p_0^\bullet = p_m(c^-)$, the first term can only be zero if $F(c_0^s\,|\,c_1)=0$, which by assumption (4) requires $c_1 > c_0^s$. The second term in (A.20) can then only be zero if the integrand $D(p_F(c_2))p_F{}'(c_2)F(c_2\,|\,c_1)$, which is continuous and non-negative, is identically zero on $[c_0^s, c_1]$. But $D(p_F(c_1)) \ge D(c^*) > 0$; by Lemma 1, $p_F{}'(c_1)>0$, and by assumption (4), $F(c_1\,|\,c_1)>0$, so this cannot be for c_2 near c_1, and (A.20) must therefore hold. Thus (A.18) holds, which, with (A.17), implies that for σ low enough (below some $\sigma^* \in (0,\sigma^{**}]$), $V(c_1)-\sigma = \xi(\sigma,c_1)-\chi(\sigma,c_1)>0$, $\forall c_1 \in [c_s^o, c^*]$. Q.E.D.

<u>Proof of Proposition 5:</u> We consider conditional distributions $F(c_2\,|\,c_1)$ satisfying assumptions (1),(2), and (4), and for which:

$$\overline{F}_2 = \sup\{F_2(c_2\,|\,c_1),\quad c^- \le c_2 \le c_1 \le c^*\}$$

is small. More precisely, we consider a family of distributions indexed by some parameter which affects F_2 continuously. For instance, if $c_i = \bar{c} + \theta + \gamma_i$, $i=1,2$, where θ is uniformly distributed on $[-a,a]$ and γ_1, γ_2 independently drawn from a uniform distribution on $[-b,b]$, with $0<a<b$, it can be shown that $\overline{F}_2 \le \frac{1}{4b}$.

We show that for distributions for which $\overline{F}_2$ is low enough:

(i) $V_m(c_1)$, the returns to search under monopoly pricing at $p=p_m(c)$, is increasing.

This defines c^*, uniquely; if $p_m(c^*)>c^*$ the equilibrium corresponds to either Proposition 2 or Proposition 3; if $p_m(c^*)<c^*$, we can construct $p_F(\cdot)$ and define $c^s \in (c^*, c^*]$. Then for $\overline{F}_2$ low enough, we show:

(ii) $V(c_1)$, the equilibrium return to search at $p=p_F(c)$, $c \ge c^s$, is increasing;

(iii) $V(c^s)>\sigma$, so that searching above p^s is optimal.

This will prove the theorem.

As before with σ, we shall make the dependence of c^*, c^s, etc., on F explicit by denoting them as c_F^*, c_F^s, etc. Since,

$$V_m'(c_1) = D(p_m(c_1))p_m'(c_1)F(c_1 \mid c_1) + \int_{c^-}^{c_1} D(p_m(c_2))p_m'(c_2)F_2(c_2 \mid c_1)dc_2 .$$

(i) will hold if for all $c_1 \in (c^-, c^*]$:

$$\overline{F}_2 \leq \frac{D(p_m(c_1))p_m'(c_1)F(c_1 \mid c_1)}{S(p_m(c^-)) - S(p_m(c_1))} . \tag{A.21}$$

We can impose (A.21) directly because the r.h.s. is a continuous, positive function of c_1, and therefore bounded away from zero. Indeed, the r.h.s. of (A.21) is strictly positive for $c_1 > c^-$, and applying L'Hopital's rule, it has limit $f(c^- \mid c^-) > 0$ at $c_1 = c^-$. Alternatively, using the convexity of $S(p)$. (A.21) is implied by,

$$\overline{F}_2 \leq \frac{D(p_m(c^*))}{D(p_m(c^-))} \min_{c_1 \in [c^-, c^*]} \left[\frac{p_m'(c_1)F(c_1 \mid c_1)}{p_m(c_1) - p_m(c^-)} \right] . \tag{A.22}$$

where similarly the term in brackets is bounded away from zero on $[c^-, c^*]$, due to (3) and L'Hopital's rule. For $c_1 \geq c_F^*$, we have:

$$V'(c_1) = D(p_F(c_1))p_F'(c_1)F(c_1 \mid c_1) + \int_{c^-}^{c_F^*} D(p_m(c_2))p_m'(c_2)F_2(c_2 \mid c_1)dc_2$$

$$+ [S(p_F^*) - S(p_F^s)]F_2(c_F^s \mid c_1) + \int_{c_F^s}^{c_1} D(p_F(c_2))p_F'(c_2)F_2(c_2 \mid c_1)dc_2$$

$$\geq D(p_F(c_1))p_F'(c_1)F(c_1 \mid c_1) - \overline{F}_2[S(p_m(c^-)) - S(p_m(c_F^*)) + S(p_F^*) - S(p_F^s) + S(p_F^s) - S(p_F(c_1))]$$

$$\geq D(p_F(c_1))p_F'(c_1)F(c_1 \mid c_1) - \overline{F}_2[S(p_m(c^-)) - S(c_1)] .$$

But,

$$p_F'(c_1) = \frac{f(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \cdot \frac{\Pi(p_F(c_1), c_1)}{\Pi_p(p_F(c_1), c_1)} \geq \frac{f(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \cdot \frac{\Pi(p_m(c_1), c_1)}{D(p_F(c_1))} = \frac{f(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \cdot \frac{\Pi_m(c_1)}{D(p_F(c_1))} .$$

so,

$$V'(c_1) \geq \frac{f(c_1 \mid c_1)F(c_1 \mid c_1)}{1 - F(c_1 \mid c_1)} \Pi_m(c_1) - \overline{F}_2[S(p^-) - S(c_1)] .$$

Thus, (ii) will hold if:

$$\overline{F}_2 \leq \min_{c \in [c_F^*, c^*]} \left[\frac{f(c \mid c)F(c \mid c)}{1 - F(c \mid c)} \right] \cdot \frac{\Pi_m(c^*)}{S(p_m(c^*)) - S(c^*)} . \tag{A.23}$$

By assumption (4), the r.h.s. is strictly positive, but both sides of (A.23) involve the distribution F. Nonetheless, if we consider a one parameter family of conditional distributions $F^{\lambda}(c_2 | c_1)$, $\lambda \in [0, \bar{\lambda}]$ which satisfy assumptions (1), (2) and (4), and such that F^{λ}, f^{λ} depend continuously on λ, with $F^0 = 0$; then as λ goes to zero, so does the l.h.s. of (A.23) whereas the minimum in the r.h.s. converges to (by L'Hopital's rule):

$$\min_{c \in [c_F^{*0}, c^*]} \frac{f^0(c | c) F^0(c | c)}{1 - F^0(c | c)} > 0,$$

since $c_F^{*0} > c^-$. Therefore, (A.23) holds for F^{λ} with λ small enough.

Finally, by (14), (iii) will hold if:

$$V(c_F^s) = [S(p_F^*) - S(p_F^s)] F(c_F^s | c_F^s) \geq \int_{c^-}^{c_F^*} D(p_m(c_2)) p_m{}'(c_2) [F(c_2 | c_F^*) - F(c_2 | c_F^s)] dc_2,$$

for which it suffices that:

$$\bar{F}_2 \leq \frac{S(p_F^*) - S(p_F^s)}{c_F^s - c_F^*} \cdot \frac{F(c_F^s | c_F^s)}{S(p^-) - S(p_F^*)}. \qquad (A.24)$$

Again, this condition involves F on both sides; moreover, it requires that the function $p_F(\cdot)$ be computed, so as to find c_F^s. Nonetheless, given a family of distribution $F^{\lambda}(c_2 | c_1)$ with the properties described above, for small λ the l.h.s. of (A.24) will be small, while the r.h.s. will be close to the finite value corresponding to F^0. This is because the equality (12) defining c_F^s always requires $p_F^s > p_F^*$, unless $\Pi(p^*, c_F^s) = 0$, i.e., $c_F^s = p^* = p_F^s$; but $c = p_F(c)$ is only possible at $c = c^*$. Thus $p_{F^0}^s = p_{F^0}^*$ would require $p_{F^0}^* = c^*$ which can be excluded by focusing (as we have) on the case where $\int_{c^-}^{c^*} D(p_m(c_2)) p_m{}'(c_2) F^0(c_2 | c^*) dc_2 > \sigma$. Q.E.D.

References

Barro, R. (1976), "Rational Expectation and the Role of Monetary Policy," Journal of Monetary Economics, 2, 1-32.

Benabou, R. (1988), "Search, Price Setting and Inflation," Review of Economic Studies, 55, 353-376.

Benabou, R. (1989), "Some Welfare Effects of Inflation," mimeo, MIT, September.

Cukierman, A. (1979), "The Relationship Between Relative Prices and the General Price Level: A Suggested Interpretation," American Economic Review, 69, 444-447.

Cukierman, A. (1982), "Relative Price Variability, Inflation, and the Allocative Efficiency of the Price System," Journal of Monetary Economics, 9, 131-162.

Cukierman, A. (1983), "Relative Price Variability and Inflation: A Survey and Further Results," Carnegie-Rochester Series on Public Policy, Autumn.

Cukierman, A. and P. Wachtel (1979), "Differential Inflationary Expectations and the Variability of the Rate of Inflation," American Economic Review, 69, 595-609.

Dana, J. (1989), "A Search-Theoretic Model of Price Stickiness," mimeo, Dartmouth College.

DeGroot, M. (1970), Optimal Statistical Decisions. New York: McGraw Hill.

Fischer, S. (1981), "Relative Shocks, Relative Price Variability, and Inflation," Brookings Papers on Economic Activity, 2, 381-441.

Fischer, S. (1984), "The Benefits of Price Stability," in Price Stability and Public Policy, The Federal Reserve Bank of Kansas City, 33-49.

Fudenberg, D. and Tirole, J. (1989) "Perfect Bayesian and Sequential Equilibrium," Harvard University Discussion Paper No. 1436, May.

Hercowitz, Z. (1981), "Money and the Dispersion of Relative Prices," Journal of Political Economy, 89, 328-356.

Lucas, R. (1973), "Some International Evidence on Output-Inflation Tradeoffs," American Economic Review, 63, 326-334.

Milgrom, P. and Weber, R. (1982) "A Theory of Auctions and Competitive Bidding," Econometrica, vol. 50, No. 5, 1089-1122.

Okun, A. (1981), Prices and Quantities: A Macroeconomic Analysis. Washington, D.C.: The Brookings Institution, 134-181.

Pagan, A., A. Hall, and P. Trivedi (1983), "Assessing the Variability of Inflation," Review of Economic Studies, 50, 585-596.

Parks, R. (1978), "Inflation and Relative Price Variability," Journal of Political Economy, 86, 79-85.

Reinganum, J. (1979), "A Simple Model of Equilibrium Price Dispersion," Journal of Political Economy, 87, 851-858.

Rosenfield, D. and R. Shapiro, (1981), "Optimal Adaptive Search," Journal of Economic Theory, 25, 1-20.

Rothschild, M. (1973), "Searching for the Lowest price When the Distribution of Prices is Unknown," Journal of Political Economy, 81, 689-711.

Taylor, J. (1981), "On the Relation Between the Variability of Inflation and the Average Inflation Rate," in K. Brunner and A. Metzler, eds., Carnegie -Rochester Conference Series on Public Policy: The Costs and Consequences of Inflation, 15, 57-85.

Vining, D. and T. Elwertowski (1976), "The Relationship between Relative Prices and the General Price Level," American Economic Review, 66, 699-708.

Date Due